THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme, Continuing Education Programme

APPLIED MATHEMATICS - LEVEL 04

PSU2182/ PCU2142/PSZ4130/PSE4182 - DESIGN AND ANALYSIS OF EXPERIMENTS

FINAL EXAMINATION 2010/2011

Duration: Two Hours.

Date: 24.06.2011	**Time: 1.30 p.m- 3.30 p.m**

Non programmable calculators are permitted. Statistical tables are provided.

Answer four questions only.

1) i. A researcher is interested in studying the effects of three diets D_1, D_2, and D_3 on the growth of mice. He decides to measure growth of mice by weight in g.

   a) Briefly explain how you would design an experiment if nine offspring born at one time to a particular mother are available.
   b) Instead of nine offspring as in part (a), if eighteen offspring for six species, each containing three offspring born at one time to a particular mother are available, how do you design the experiment?

   ii. Briefly explain the importance of the following basic principles in designing experiments.
   a) Randomization
   b) Replication

2) A researcher wants to determine whether a new computer program helps students in grade 9 to learn mathematics. He selected 21 male students among the best performers from the 9th grade in a school and divided them into three groups of seven randomly. Students assigned to group I received no special mathematics training other than regular class room instruction. Students in group II received extra tutoring from the teacher and the students in group III were tutored using the computer program. After two weeks the researcher gave all the students a general mathematics test. Their test marks are shown in the table below.

	Group I	**Group II**	**Group III**
	24	30	52
	23	23	50
	22	22	44
	23	24	62
	18	18	60
	18	16	75
	16	21	55
Total	**144**	**154**	**398**

Grand total = 696

Total uncorrected sum of squares = $\sum y^2$ =29766

i. Write down the hypotheses to be tested.
ii. Test the hypotheses stated in part (i) at 5% significance level. Clearly state your findings.
iii. Construct a 95% confidence interval for the difference between the mean test marks of group I and group III. Interpret the result.

3) A seed company wants to conduct an experiment to compare the yield of four varieties of wheat. Five fields are available for the study and each field was sub divided into four homogeneous plots of equal size. In each field, each variety was randomly assigned to a plot. At the end of the season, yield in bushels was recorded for each plot.

i. Name the design of the above experiment.
ii. Write down a model for the response measured on a randomly chosen plot. Clearly explain the notation you use.
iii. Some part of the ANOVA table is given below. Complete the ANOVA table and report your conclusions. Make your decisions at 5% significance level.

Source of Variation	Sum of squares
Wheat	636
Field	504
Total	1200

4) i. Explain what is meant by a factorial experiment. Explain an advantage of a factorial experiment over the single factor experiment?

ii. A scientist suspects that the yield of a chemical process is influenced by the pressure and the temperature. He selected three levels of each factor and performed two replicates with each combination of pressure and temperature. The yield data (in grams) are presented in the following table.

Temperature	Pressure		
	200	215	230
150	90.4 90.2	90.7 90.6	90.2 90.4
160	90.1 90.3	90.5 90.6	89.9 90.1
170	90.5 90.7	90.8 90.9	90.4 90.1

Grand total = 1627.4

a) Write down a model for the response measured on a randomly chosen process. Clearly explain the notation you use.
b) Consider the ANOVA table that can be used to address the experimenter's objective. Compute the total sum of squares.
c) In completing the ANOVA table referred in part (b), the total sum of squares will be decomposed into several sums of squares. Name these components and write down the corresponding degrees of freedom. You need not compute the values of these components.

5) The effect of five different ingredients (A, B, C, D, and E) on the reaction time of a chemical process was studied. Each batch of new material was only large enough to permit five runs to be made. Furthermore, each run required approximately 1 ½ hours, so only five runs could be made in one day. The experimenter decided to use the Lattin square design with 5 batches and 5 days. The data obtained are given below.

Batch	**Day**					**Total**
	1	**2**	**3**	**4**	**5**	
1	8(A)	7(B)	1(D)	7(C)	3(E)	**26**
2	11(C)	2(E)	7(A)	3(D)	8(B)	**31**
3	4(B)	9(A)	10(C)	1(E)	5(D)	**29**
4	6(D)	8(C)	6(E)	6(B)	10(A)	**36**
5	4(E)	2(D)	3(B)	8(A)	8(C)	**25**
Total	**33**	**28**	**27**	**25**	**34**	**147**

Total uncorrected sum of squares = $\sum y^2 = 1071$

Treatment total	**A**	**B**	**C**	**D**	**E**
	42	28	44	17	16

i. Test whether there is a significant difference between the effects of 5 ingredients. Make your decision at 5% significance level. Clearly state your findings.
ii. Find the Least Significant Difference (LSD) for comparing any two treatment means. Use 5% significance level.
iii. Using part (ii) or otherwise test whether ingredients C and E are equally effective. Use 5% level of significance. Clearly state your findings.

6) A 2^3 factorial experiment was carried out to investigate the effects of three factors on the life time (in hours) of a machine tool. The factors considered were cutting speed (A), tool geometry (B) and cutting angle (C). Each factor was applied at two levels and 24 machine tools were made under the 8 treatment combinations using a completely randomized design with equal number of replicates. The treatment total for each treatment combination is given below.

Treatment combination	**(1)**	**a**	**b**	**c**	**ab**	**ac**	**bc**	**abc**
Life time	78	104	119	127	148	113	164	127

A part of the Analysis of Variance (ANOVA) table related to this study is given below.

Source of Variation	**Degrees of freedom**	**Sum of squares**
Treatment	7	1612.7
Error	16	482.7
Total	23	2095.4

i. Using Yate's algorithm, compute all the main effects and interactions.
ii. Extend the given analysis of Variance (ANOVA) table to test the significance of all the main effects and interactions. Use 5% level of significance. Clearly explain your conclusions.