THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme, Continuing Education Programme

APPLIED MATHEMATICS - LEVEL 04

PSU2182/ PCU2142/PSZ4130- DESIGN AND ANALYSIS OF EXPERIMENTS

CLOSED BOOK TEST 2010/2011

Duration: One and Half Hours.

Date: 30.04.2011	Time: 11.00 a.m- 12.30 p.m

Answer all questions.

1. A medical researcher is interested in determining which of the four different blood pressure pills are most effective in controlling a person's blood pressure. The researcher gave each blood pressure pill to five different people over a period of one month and recorded the average blood pressure loss. The data are presented in the accompanying table.

Blood pressure pill	**Average systolic blood pressure loss (in mm Hg)**				
A	22	11	17	15	10
B	12	16	18	13	15
C	9	11	5	8	6
D	16	3	9	10	11

Grand total = $\sum x = 237$ Total uncorrected sum of squares = $\sum x^2 = 3231$

i) Construct an analysis of variance (ANOVA) table and test whether there is a significant difference between treatment means at 5% significance level. Clearly state your findings.

ii) Estimate the least significant difference for comparing any two treatment means. Use 5% significance level.

iii) Using part (ii) or otherwise determine whether there is a significant difference between the blood pressure loss when blood pressure pill A and C are applied. Use 5% significance level. Clearly state your findings.

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2. Three different washing solutions are being compared to study their effectiveness on bacteria growth in 5 gallon milk containers. The analysis is done in a laboratory, and only three trials can be run on any day. Since days could represent a potential source of variability, the experimenter decided to use a randomized complete block design. Observations are taken for four days and the data are as in the following table.

Solutions	**Days**			
	1	**2**	**3**	**4**
1	13	22	18	39
2	16	24	17	44
3	5	4	1	22

Grand total = $\sum x = 225$ Total uncorrected sum of squares = $\sum x^2 = 6081$

i) Write down a model for the response measured on a randomly chosen milk container. Clearly describe the notation you use.

ii) Estimate the difference between the means of solution 1 and solution 3.

iii) Give an estimate for the standard error of the estimate given in part (ii).

3. In an experiment to study the effect of fertilizer on the yield of certain type of paddy, two amounts of fertilizer (5lbs and 10lbs) along with the control (that is no fertilizer) were applied to 12 plots three months prior to harvesting. These 12 plots of land were randomly divided into three groups of four each. Plots in each group were treated with one of the three fertilizers. Soil conditions of the plots, amount of fertilizers used and the plot sizes were quite similar. The observed yields per plot (in bushels) are presented in the accompanying table.

i) State whether the following statements are true or false. In each case, give reasons for your answer.

   a) The design used in this study is a completely randomized design.

   b) One of the weaknesses in the design used in this study is that there are no replicates.

c) The treatment structure used in this study is a three-way treatment structure.

ii) Write down a model for the response measured on a randomly selected plot. Clearly describe the notation you use.

iii) Suggest a meaningful independent comparison to compare the effects between

a) Fertilizer and Control

b) Two fertilizers A and B

iv) How many linearly independent treatment mean comparisons are possible?

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Closed Book Test – 2010/2011 – PSU2182/PCU2142
Answer Guide

1) (i) H_O: There is no significant difference between treatment means.
H_1: There is a significant difference between treatment means.

Correction Factor (CF) = $(237)^2/20$ =2808.45

Total SS = $\sum_{ij} y_{ij}^2$ - CF = 3231 – 2808.45 = 422.55

Treatment SS = $\sum_{i=1}^{4} \frac{y_i^2}{5} - CF = \frac{(75)^2+(74)^2+(39)^2+(49)^2}{5} - 2808.45 = 196.15$

Error SS = Total SS - Treatment SS = 422.55 - 196.15 = 226.40

SOV	SS	Df	MS	F value
Treatment	196.15	3	65.38	4.62
Error	226.4	16	14.15	
Total	422.55	19		

$F_{tab} = F_{3,16,0.05} = 3.29$
Since $F_{cal}(4.62) > F_{tab}(3.29)$, the null hypothesis is rejected at 5% level of significance. This provides strong evidence that treatment effects are different. Hence we conclude that sufficient evidence is available at 5% significance level to claim that there is significant difference between the effects of blood pressure pills on blood pressure loss.

(ii) Least Significant Difference (LSD) = $t_{\omega\theta}\sqrt{2S^2/n} = 2.13\sqrt{2(14.15)/5} = 5.07$

(iii) **1st method**

$$\bar{Y}_A = \left(\frac{22+11+17+15+10}{5}\right) = 15$$

$$\bar{Y}_C = \left(\frac{9+11+5+8+6}{5}\right) = 7.8$$

$$\left|\bar{Y}_A - \bar{Y}_C\right| = 7.2$$

Since $\left|\bar{Y}_A - \bar{Y}_C\right|$>LSD (5.07), we find that there is a significant difference between the blood pressure loss when blood pressure pill A and C are applied.

2nd method

95% confidence interval for the difference between treatment means A and C

$$\bar{Y}_A - \bar{Y}_C \pm t_{e,\theta}\sqrt{S^2\left(\frac{1}{r_A}+\frac{1}{r_C}\right)} \qquad t_{16,0.05} = 2.13$$

$$= (15-7.8) \pm 2.13\sqrt{14.15\left(\tfrac{1}{5}+\tfrac{1}{5}\right)} = (2.13, 12.27)$$

Since the interval does not contain value zero, we have enough evidence to assume that the true difference between effectiveness of treatment A and C is non-zero at 5% significance. That mean treatment A and C are not equally effective.

2) (i) $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ i=1,2,3 j=1,2,3,4

Y_{ij} – Observation taken on the i^{th} solution in j^{th} day

μ - General mean of response

α_i - Effect of i^{th} solution

β_j - effect of j^{th} day

ε_{ij} - Random error

(ii) $\bar{Y}_1 - \bar{Y}_3 = \left(\frac{13+22+18+39}{4}\right) - \left(\frac{5+4+1+22}{4}\right) = 15$

(iii) Correction Factor (CF) = $(225)^2/12 = 4218.75$

Total SS = $\sum_{ij} y_{ij}^2$ - CF = 6081 – 4218.75 = 1862.25

Treatment SS = $\sum_{i=1}^{3} \frac{y_{i.}^2}{4} - CF = \frac{(92)^2+(101)^2+(32)^2}{4} - 4218.75 = 703.5$

Block SS = $\sum_{j=1}^{4} \frac{y_{.j}^2}{3} - CF = \frac{(34)^2+(50)^2+(36)^2+(105)^2}{3} - 4218.75 = 1106.92$

Error SS = Total SS - Treatment SS - Block SS = 1862.25 - 703.5 - 1106.92 = 51.83

Error mean square (S^2) = 51.83/6 = 8.64

Estimate for Standard error = $\sqrt{2S^2/r} = \sqrt{2*8.64/4} = 2.08$

3) (i) a) True

These 12 plots are homogeneous. Because soil conditions of the plots, amount of fertilizer used and the plot sizes were quite similar. There is only one factor and no

blocking variable. Treatments are assigned to the experimental units completely at random. Hence this design is a completely randomized design.

b) False

There are 4 replicates for each treatment.

c) False.

There is only one factor. Hence the treatment structure used in this study is one-way treatment structure.

(ii) $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ i=1,2,3 j=1,2,3,4

Y_{ij} - Observation for the j^{th} plot to which fertilizer i is applied.

μ - General mean of response

α_i - Effect of i^{th} fertilizer

ε_{ij} - Random error

(iii) a) Let L_1 be the comparison between fertilizer and control.

$L_1 = 2T_C - T_{F5} - T_{F10}$

b) Let L_2 be the comparison between two amounts of fertilizer.

$L_2 = T_{F10} - T_{F5}$

Here T_C, T_{F5} and T_{F10} denote the treatment total for the control, fertilizer 5lbs and fertilizer 10lbs respectively.

(iv) Since there are 3 treatments, two linearly independent treatment mean comparisons are possible.

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