MPZ 3230 – Engineering Mathematics I
Assignment No. 04 –Academic Year - 2007

Answer all questions

(01) (i) Evaluate $\int_0^2 y dx$ from the following table using trapezoidal rule.

X	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Y	1.21	1.37	1.46	1.59	1.67	2.31	2.91	3.83	4.01	4.79	5.31

(ii) State Simpson's rule,

A car laps a race track in 84s. The speed of the car at each 6 second interval is determined using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table:

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track? You may use approximation and if any state them.

(02) State Taylor series expansion,

Using the Taylor series expansion of $\frac{1}{1+x}$, show that,

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots\ldots\ldots$ (-1 < x < 1)

And given $\tan\theta = \frac{1}{2}$, find an approximate value for θ

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(03) The following table gives a grouped frequency distribution of the lifetimes of a random sample of 427 electric bulbs taken from the output of a particular factory:

Lifetime(in months)	Number of bulbs
15-19	66
20-24	65
25-29	56
30-34	50
35-39	42
40-44	37
45-49	35
50-54	30
55-59	24
60-64	22

(a)Estimate, to one decimal place,

(i) The mode

(ii) The median
(iii)the lower quartile,
(iv)the upper quartile,
Of these lifetimes.

(b)Evaluate, to one decimal place,
(i) The mean,
(ii) The standard deviation,
(iii) The coefficient of skewness,
Of this distribution.
What is the shape of this distribution?

(04) (i)State the Bayes' theorem.

A manufacturing company buys 80% of its components from supplier X and 20% from supplier Y. Last year, 5% of the components supplied by supplier X were defective and 1% of the components supplied by supplier Y were defective. What proportions of the company's defective components were supplied by supplier X

(05). (i) A, B, C are three events write $P(A \cup B \cup C)$ in simple probability terms.
(ii) Using a Venn diagram show that,
$(A \cap B') \cup (A \cap B) = A$
Hence deduce that if A and B are independent, Probabilities relating to events A, B and C are as follows.

$P(A) = \frac{1}{5}$, $P(B) = \frac{1}{6}$, $P(A \cap C) = \frac{1}{20}$, $P(B \cup C) = \frac{3}{8}$.

Evaluate P(C) and show that B and c are independent.

(06) The weight of potatoes consumed by a student in a week is a random variable X kg. A suggested model for the probability density function for x is

$$f(x) = \begin{cases} kx(1-x)e^{x} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k.

(i) Find the cumulative distribution function of X.
(ii) Find the probability that a student eats between 0.5kg and 0.75kg of potatoes in a week
(iii) Find the mean and the variance of X.

Please send your assignment **on or before** **09.01.2008** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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Model answer – Assignment No. 04
MPZ 3230 – Academic Year 2007

(01).(i)

x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.21	1.37	1.46	1.59	1.67	2.31	2.91	3.83	4.01	4.79	5.31

Evaluate $\int_0^2 ydx$

By using Trapezoidal rule,

$I = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots y_{n-1})]$

$h = 0.2 - 0 = 0.2$

Then;

$I = \frac{h}{2}[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)]$

$= \frac{0.2}{2}[(1.21 + 5.31) + (1.37 + 1.46 + 1.59 + 1.67 + 2.31 + 2.91 + 3.83 + 4.10 + 4.79)]$

$I = 5.44$

$\int_0^2 ydx = 5.44$

(ii) Simpson rule (I)

$I = \int_a^b f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \ldots y_{n-2}) + 4(y_1 + y_3 + \ldots y_{n-1})]$

$h = (x_1 - x_0)$

	0	6	12	18	24	30	36	12	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

$h = 6$

$I = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4 + \ldots y_{n-2}) + 4(y_1 + y_3 + \ldots y_{n-1})]$

$= \frac{6}{3}[(124 + 123) + 2(148 + 147 + 121 + 99 + 78 + 104)$

$+ 4(134 + 156 + 133 + 109 + 85 + 89 + 116)]$

$= 2[247 + 2(697) + 4(822)]$

$I = 9858$ *feet*

$\therefore$ *Long is the track* $= 9858$ *feet*

(02). Tailor Series

$$f(x) = f(0)+\frac{f'(0)x}{1!}+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+.....$$

Show,

$$f(x) = \frac{1}{1+x} \quad \text{Tailor expansion,}$$

$$f(x) = \frac{1}{1+x} \qquad f(0) = 1$$

$$f'(x) = \frac{-1}{(1+x)^2} \qquad f'(0) = -1$$

$$f''(x) = (-1)(-2)(1+x)^{-3} \qquad f''(0) = 2$$

$$= \frac{2}{(1+x)^3}$$

$$f'''(x) = 2(-3)(1+x)^{-4} \qquad f'''(0) = -6$$

$$= \frac{-6}{(1+x)^4}$$

$$\frac{1}{1+x} = f(0)+\frac{f'(0)x}{1!}+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+.....$$

$$\frac{1}{1+x} = 1+(-1)x+2\frac{x^2}{2!}+(-6)\frac{x^3}{3!}+.....$$

$$\frac{1}{1+x} = 1-x+x^2-x^3+.....$$

Substituting $x = x^2$

$$\frac{1}{1+x^2} = 1-(x^2)+(x^2)^2-(x^2)^3+.....$$

$$\int\frac{1}{1+x^2} = 1-x^2+x^4-x^6+x^8-x^{10}.....$$

$$\tan^{-1}x = \int\left(1-x^2+x^4-x^6+x^8-x^{10}....\right)dx$$

$$= x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}+.....$$

$$\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+..... \qquad (-1<x<1)$$

Given $\tan\theta = 1/2$

$$\theta = \tan^{-1}\left(1/2\right)$$

Apply above prove,

$$\tan^{-1}\left(1/2\right) = 1/2-\frac{(1/2)^3}{3}+\frac{(1/2)^5}{5}-\frac{(1/2)^7}{7}+....$$

$$= 0.5-0.042+0.00625-0.00112$$

$$= \underline{0.46313}$$

(03)

Life Time	(fi) Number of bulbs	True class Interval	(ci) Cumulative frequency	Mid points	$fimi$	$di=\frac{mi-42}{5}$	$fidi$	$fidi^2$
15-19	66	14.5-19.5	66	17	1122	-5	-330	1650
20-24	65	19.5-24.5	131	22	1430	-4	-260	1040
25-29	56	24.5-29.5	187	27	1512	-3	-168	504
30-34	50	29.5-34.5	237	32	1600	-2	-100	200
35-39	42	34.5-39.5	279	37	1554	-1	-42	42
40-44	37	39.5-44.5	316	42	1554	0	0	0
45-49	35	44.5-49.5	351	47	1645	1	35	35
50-54	30	49.5-54.5	381	52	1560	2	60	120
55-59	24	54.5-59.5	405	57	1368	3	72	216
60-64	22	59.5-64.5	427	62	1364	4	88	352
total					14709		-645	4159

(a).I. Mode

$$= L+\left(\frac{d_1}{d_1+d_2}\right)w$$

$$= 14.5+\left(\frac{66}{66+1}\right)5$$

$$= 19.4$$

$d_1 = 66-0 \quad = 66$

$d_2 = 66-65 \quad = 1$

$L \ = 14.5, \quad W = 5$

II. Median

$$= L+\frac{\left[\frac{n}{2}-F_1\right]}{f}\ w$$

$$= 29.5+\frac{(213.5-187)}{50}\times 5$$

$$= 32.1$$

$L = 29.5$

$n/2 = \frac{427}{2} = 213.5$

$F_1 = 187$

$f = 50$

$w = 5$

III. Lower quartile

$$= L+\left[\frac{\frac{n}{4}-F_1}{f}\right]w$$

$$= 19.5+\frac{[106.75-66]}{65}\times 5$$

$$= 22.6$$

$L = 19.5$

$n/4 = \frac{427}{4} = 106.75$

$F_1 = 66$

$f = 65$

$w = 5$

IV. Upper quartile $= L + \dfrac{\left[\frac{3n}{4} - F_1\right]}{f} \times w$

$= 44.5 + \dfrac{(326.25 - 316)}{35} \times 5$

$= 45.1$

$L = 44.5$

$\dfrac{3n}{4} = \dfrac{427 \times 3}{4}$

$= 320.25$

$F_1 = 316$

$f = 35$

$w = 5$

(b) (I) Mean, (method 1)

$\overline{X} = A + C\overline{d}$

$= A + C\dfrac{(fidi)}{n}$

$= 42 + 5 \times \dfrac{(-645)}{427}$

$= 34.4473$

$= 34.4$

(Method 2)

Mean, $= \sum_{x=1}^{n} \dfrac{fimi}{n}$

$= \dfrac{14709}{427}$

$= 34.4$

(II) Standard deviation $= S$

$S = C\sqrt{\dfrac{\sum_{i=1}^{k} f_i d_i^2}{n} - (\overline{d})^2}$

$= 5\sqrt{\dfrac{4159}{427} - (-1.51)^2}$

$= 13.7$

$\overline{d} = \dfrac{\sum fidi}{n}$

$= \dfrac{-645}{427}$

$= -1.51$

(III) Skewness $= Sk_p$

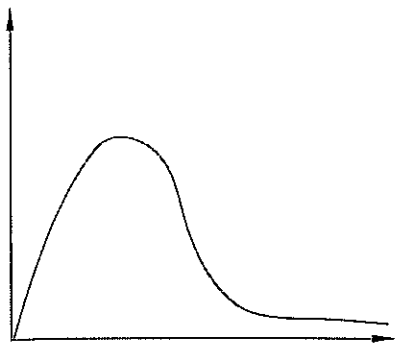
$Sk_p = \dfrac{\text{mean - mode}}{\text{Standard deviation}}$

$= \dfrac{34.4 - 19.4}{13.7}$

$= 1.09489$

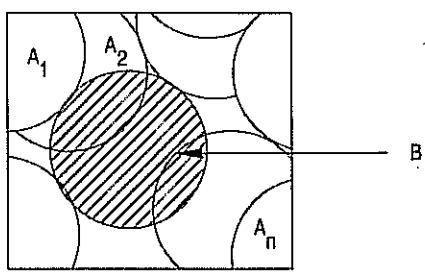
$= 1.1$

Positive skewness and shape is,



(04).

(i) Baye's Theorem:



If $A_1, A_2, ...A_n$ are mutually excusive and collectively exhaustive events of a sample space. S and B is given event in S then,

$$P\left(Ak/B\right) = \frac{P\left(B/Ak\right).P(Ak)}{\sum_{i=1}^{n}\left[P\left(B/Ai\right).P(Ai)\right]} \quad , for \quad k = 1,2....,k$$

Supplier x	Supplier y
Supply $p(x) = 80\%$	$p(y) = 20\%$
Defective $p\left(D/x\right) = 5\%$	$p\left(D/y\right) = 1\%$

Let company's defective components were supplied by supplier x is $p\left(x/D\right)$

Apply Baye's Theorem,

$$p\left(x/D\right) = \frac{p\left(D/x\right)p(x)}{p\left(D/x\right)p(x) + p\left(D/y\right)p(y)}$$

$$= \frac{0.05 \times 0.8}{0.05 \times 0.8 + 0.01 \times 0.12}$$

$$= \frac{20}{21}$$

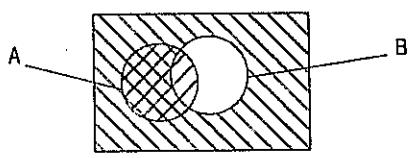
$= 0.952380952$

Company's defective components were supplied by

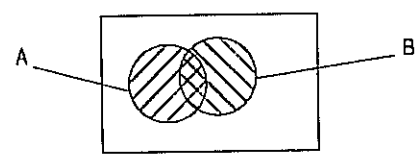
Suppler x is $= 95\%$

(05). By Using Venn Diagram,

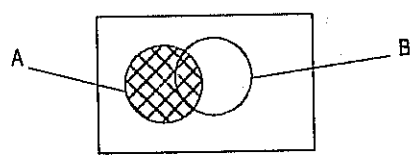
$(A \cap B') \cup (A \cap B) = A$



$A \cap B'$



$A \cap B$



$(A \cap B') \cup (A \cap B)$

If A and C are independent

$\therefore \quad P(A \cap C) = P(A) \times P(C)$

$P(A) = 1/5, \quad P(B) = 1/6, \quad P(A \cap C) = \frac{1}{20}, \quad P(B \cap C) = \frac{3}{8}$

$P(A \cap C) = P(A) \times P(C)$

$1/20 = 1/5 \times P(C)$

$P(C) = 1/4$

$\underline{k = 3.55}$

There fore probability density function is $f(x)$

There fore $f(x) = \begin{cases} 3.55x(1-x)e^x dx & : 0 \le x \le 1 \\ 0 & : otherwise \end{cases}$

(i) Cumulative distribution function is $F(x)$

For $o \le t \le 1$ $F(t) = \int_0^t 3.55\ x(1-x)e^x dx$

$$F(t) = 3.55\int_0^t x(1-x)e^x dx$$

$$= 3.55\left[\int_0^t xe^x dx - \int_0^t x^2 e^x dx\right]$$

$$= 3.55\left[\left[xe^x\right]_0^t - \left[e^x\right]_0^t - \left[x^2e^x\right]_0^t + \int_0^t e^x 2x dx\right]$$

$$= 3.55\left[\left[xe^x\right]_0^t - \left[e^x\right]_0^t - \left[x^2e^x\right]_0^t + \left[2xe^x\right]_0^t - \left[2e^x\right]_0^t\right]$$

$$= 3.55\left[te^t - e^t + e^0 - t^2e^t + 2te^t - 2e^t + 2e^0\right]$$

$$= 3.55\left[3te^t - 3e^t - t^2e^t + 3\right]$$

$$F(t) = \frac{1}{(3-e^t)}\left\{3\left(te^t - e^t + 1\right) - t^2e^t\right\}$$

$$F(t) = \frac{1}{(3-e^t)}\left\{3\left(e^1 - e^1 + 1\right) - e^1\right\} = \frac{(3-e)}{(3-e)} = 1$$

$$F(t) = \frac{1}{(3-e^t)}\left\{3\left(0 - e^0 + 1\right) - 0\right\} = 0$$

Cumulative distribution function is,

$$F(t) = \begin{cases} & x \le 0 \\ 3\left(te^t - e^t + 1\right)t\ e^t & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

(ii) $P(0.5 \le x \le 0.75) = \int_{0.5}^{0.75} 3.55x(x-1)e^x dx$

$$= 3.55\left[\int_{0.5}^{0.75} xe^x dx - \int_{0.5}^{0.75} x^2 e^x dx\right]$$

$$= 3.55\left[\left[xe^x\right]_{0.5}^{0.75} - \left[e^x\right]_{0.5}^{0.75} - \left[x^2e^x\right]_{0.5}^{0.75} + \int_{0.5}^{0.75} 2e^x x dx\right]$$

$$= 3.55\left[\left[xe^x\right]_{0.5}^{0.75} - \left[e^x\right]_{0.5}^{0.75} - \left[x^2e^x\right]_{0.5}^{0.75} + \left[2xe^x\right]_{0.5}^{0.75} - 2\left[xe^x\right]_{0.5}^{0.75}\right]$$

$$= 3.55\left[\left[3xe^x\right]_{0.5}^{0.75} - \left[3e^x\right]_{0.5}^{0.75} - \left[x^2e^x\right]_{0.5}^{0.75}\right]$$

$$P(B\cup C) = P(B)+P(C)-P(B\cap C)$$
$$3/8 = 1/6+1/4-P(B\cap C)$$
$$P(B\cap C) = \frac{10}{24}-\frac{9}{24}$$
$$P(B\cap C) = \frac{1}{24}$$

We know that, $P(B)=1/6$, $P(C)=1/4$,

If A and B independent,

$$P(B\cap C) = P(B)\times P(C)$$
$$P(B)\times P(C) = 1/6\times 1/4$$
$$= \frac{1}{24}$$

$\therefore$ B and C are independent

(06).
$$= \begin{cases} kx(1-x)e^x & 0\le x\le 1 \\ 0 & otherwise \end{cases}$$

$$\int_0^1 f(x)dx = 1$$

$$\int_0^1 kx(1-x)e^x\,dx = 1$$

$$k\left\{\int_0^1 xe^x dx - kx^2e^x\,dx\right\} = 1$$

$$k\left\{\left[xe^x\right]_0^1 - \int_0^1 e^x dx - \left[x^2e^x\right]_0^1 + \int_0^1 e^x 2x d\quad dx\right\} = 1$$

$$k\left\{e^1-\left[e^1-e^0\right]-\left[e^1\right]+\left[2xe^x\right]_0-2\int_0^1 e^x dx\right\} = 1$$

$$k\left\{e^0-e^1+2e^1-2\left(e^1-e^0\right)\right\} = 1$$

$$k\left\{e^0+e^1-2e^1+2e^0\right\} = 1$$

$$k\left\{3e^0-e^1\right\} = 1$$

$$k = \frac{1}{\left(3e^0-e^1\right)} = \frac{1}{\left(3-e^1\right)}$$