MPZ 3230 – Engineering Mathematics I
Assignment No. 03 –Academic Year - 2007

Answer all questions

(01). Define the angle between

(a) Two skew straight lines.
(b) A straight line and a plane.
(c) Two planes.

A right regular pyramid of height **'a'** stands on a regular hexagonal base ABCDEF of side **'a'**, the vertex of the pyramid being 0. Determine

(i) The angle between OA and the sides of the base.
(ii) The angle between OA and the plan OCE.
(iii) The angle between the planes OAB and OBC.

(02) State the theorem of 3 perpendiculars.

The triangles ABC, ABD are in perpendicular planes and are right-angled at A and B respectively is the mid-point of AB and Q the mid-point of CD. Show that PQ is perpendicular to AB.

(03).A trough of length L has a cross section in the shape of a semicircle with radius. When filled with water to within a distance h of the top, the volume V of water is,

$$V = L\left[0.5\Pi r^2 - r^2 \arcsin(h/r) - h\left(r^2 - h^2\right)^{\frac{1}{2}}\right]$$

Suppose L = 10 feet, r =1 foot and V =12.4 cubic feet. Use the Bisection method to find the depth of water in trough to within 0.01 feet. (Value has within 0.1, 0.18 interval)

(04)

(a) $x^3 + x^2 + x + 7 = 0$, Use the method of false position to obtain a root, **correct to three decimal places**

(b) $\cos x = 3x - 1$, Use the iterative method, find the roots **correct to four significant figures.**

(c) $x^3 - 3x^2 + 7x - 8 = 0$ Using Newton-Raphson method, find a root **correct to three decimal places.**

(05) (a) Solve the following equation by using **Gauss elimination method.** (*Using matrix form*)

$$3x + 4y - z = 8$$
$$-2x + y + z = 3$$
$$x + 2y - 2 = 2$$

(b) Solve the following equation by using **Jacobi method.**

$$2x - 3y + z = -1$$
$$x + 4y + 5z = 25$$
$$3x - 4y + z = 2$$

(c) Solve the following equation by using **Gauss-Seidel method**

$$3x + y + 2z = 16$$
$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$

(06). (i) (a) Write down Newton forward interpolation formula, Newton backward interpolation formula

(b) The table given the values of tan x for $0.10 \leq x \leq 0.30$.

X	0.10	0.15	0.20	0.25	0.30
Y= tan x	0.1003	0.1511	0.2027	0.2553	0.3093

Find,

(a) tan 0.12

(b) tan 0.26 values,

Using **suitable** Newton interpolation formula and write down why it using.

(ii) The following data give the percentage of criminals for different age groups:

Age(less than x)	25	30	40	50
% of criminals	52	67.3	84.1	94.4

Using Lagrange's interpolation formula, find the percentage of criminals under the age of 35.

Please send your assignment **on or before** **28.11.2007** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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MPZ 3230
Model answer No. 03 – Academic Year 2007

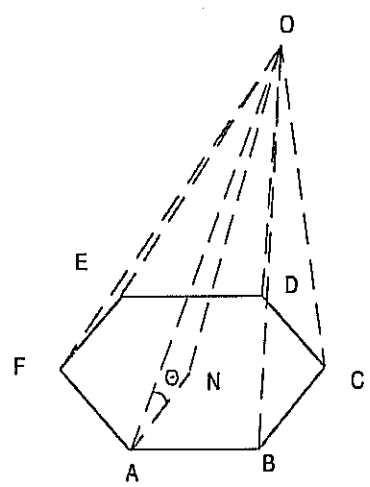
(01)

(a) Angle between two skew straight lines:-
It is defined as the angle between two lines parallel to given skew lines at any point.

(b) Angle between a straight line and a plane:-
The line and its projection are coplanar and so meet in a point on the plane (identically, note that point is its own projection). The angle between line and its projection is defined to be the angle between the line and the plane.

(c) Angle between two planes:-
Two planes which in a line are said to from a dihedral angle. Such as angle is measured by the angle between the normal's to the planes.

(i)



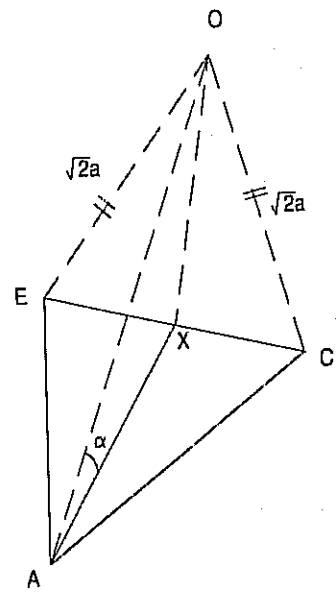
N is the centre of the base.
ON is normal to the base and $ON = a$, $AN = a$

(i) The angle between OA and the sides of the base is θ

$$Tan\theta = \frac{ON}{AN} = \frac{\theta}{a} = 1$$

$$\therefore \theta = \pi/4$$

(ii)



$$\cos\ o\hat{c}x = \frac{\sqrt{3}\,a/2}{\sqrt{2}\,a}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}}$$

$$ox = \sqrt{\left(\sqrt{2}a\right)^2 - \left(\frac{\sqrt{3}a}{2}\right)^2}$$

$$ox = \frac{\sqrt{5}a}{2}$$

AEC is equilateral triangle with $AE = \sqrt{3}\,a$

X is the mid point of EC

$\therefore A \perp EC$

$AX = \sqrt{3}\,a\sin 60$

$$= \frac{3a}{2}$$

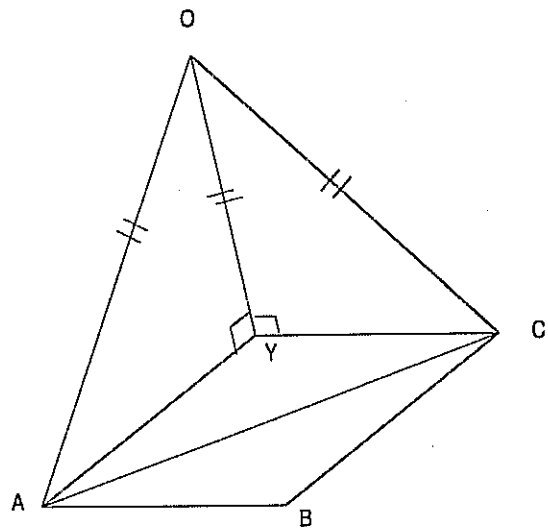
Orthogonal projection of OA on the plane OCE is OX,

Let Angle between OA between OA and $O\hat{C}X$ is α

$$Tan\alpha = \frac{OX}{AX} = \frac{\sqrt{5}a/2}{3a/2} = \frac{\sqrt{5}}{3}$$

$$\alpha = 37^{\circ}$$

(iii)



We selected point Y on the Line OB.
Such that

$AY \perp OB$

$\therefore CY \perp OB$

$\therefore \quad OAB\Delta \equiv OBC\Delta$

$OA = OB = \sqrt{2}\,a, \qquad AB = a$

$\therefore \quad O\hat{A}B\ \Delta$ Apply cosine rule,

$$OB^2 = OA^2 + AB^2 - 2.OA.AB\cos A$$

$$2a^2 = 2a^2 + a^2 - 2\sqrt{2}\,a^2\cos A$$

$$\cos A = \frac{1}{2\sqrt{2}} = \cos B$$

$$AY = AB\sin B$$

$$= a \times \frac{\sqrt{7}}{2\sqrt{2}}$$

$$YC = \frac{\sqrt{7}}{2\sqrt{2}}a$$

$$AC = \sqrt{3}a$$

From AYC tangle,

$$AC^2 = AY^2 + YC^2 - 2AY \cdot YC \cos A\hat{Y}C$$

$$3a^2 = \left(\frac{7a^2}{8}\right) \times 2 - 2\frac{\sqrt{7}a}{2\sqrt{2}} \cdot \frac{\sqrt{7}a}{2\sqrt{2}} \cos A\hat{Y}C$$

$$\cos A\hat{Y}C = -\frac{\left(3a^2 - \frac{7a^2}{4}\right)}{\frac{7a^2}{4}}$$

$$\cos A\hat{Y}C = -\frac{\left(\frac{5}{4}\right)}{\left(\frac{7}{4}\right)}$$

$$A\hat{Y}C = \cos^{-1} \cdot \left(-\frac{5}{7}\right)$$

$$= 136^\circ$$

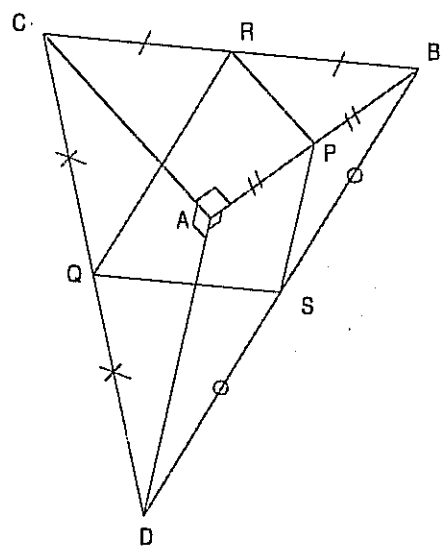
(02). P is point outside the plane α. PO is normal to α and PL is perpendicular to L.
Theorem 15: Theorem of three perpendiculars.

P is any point not lying on a plane α and l is a line lying on α.

(i) If O and M are the feet of the perpendiculars from P to α and l respectively, then OM is perpendicular to l.

(ii) If O is the foot of the perpendicular from p to α and if M is the foot of the perpendicular from O to l, then PM is Perpendicular to l.

(ii) If M is the foot of the perpendicular from P to l and if O is the foot of the perpendicular from P to the line on α through M and normal to l, then PO is Perpendicular to α.



$PS \perp AB$
$PR \perp AB$ $\Big\}$ $AB \perp PSR$ plane

$\therefore AB \perp RS$

$AB \perp CD$
$AB \perp AC$ $\Big\}$ $AB \perp ACD$ plane

$\therefore AQ \perp AB$
$AQ \perp AP$ $\Big\}$ $AB \perp APQ$ plane

If $AB \perp APQ$ plan

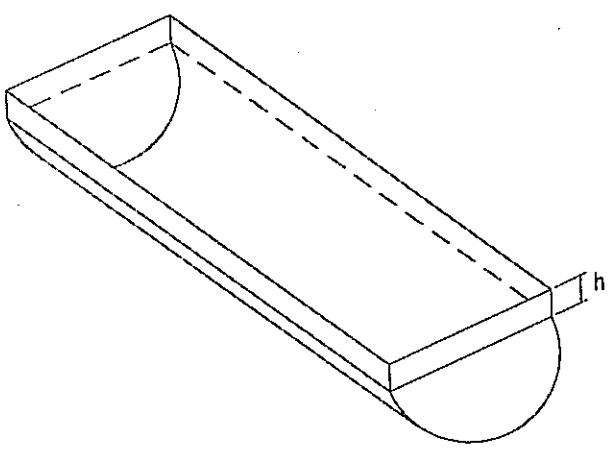
$AB \perp PQ$.

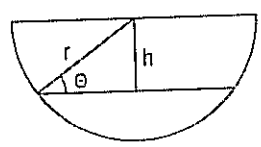
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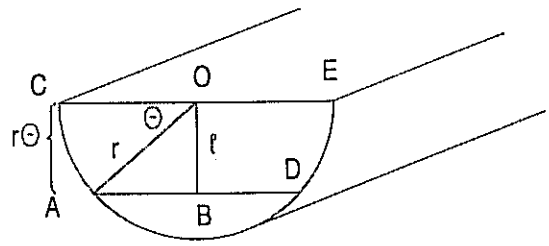
(03). A through of Length L has a cross section in the shape of a semicircle with radius. When filled with water to within a distance h of the top, the volume v of water is

$$V = L\left[0.5\pi r^2 - r^2 \arcsin\left(h/r\right) - h\left(r^2 - h^2\right)^{1/2}\right]$$

Suppose $L = 10$ feet, $r = 1$ foot and $V = 12.4$ cubic feet. Use the Bisection method to find the depth of water in through to within 0.01 feet.







$$\sin\theta = \frac{h}{r}$$

$$\theta = \sin^{-1}\left(h/r\right)$$

$$\Delta\ OAB\ = \tfrac{1}{2}\left(r^2 - h^2\right)^{1/2} h$$

$$\Delta\ OAC\ = \tfrac{1}{2}\ r.\, r\theta$$

$$= \tfrac{1}{2}\ r^2\theta - \tfrac{1}{2} r^2 \sin^{-1}\left(h/r\right)$$

$$A^1\quad [ABDEOCA] \quad = 2\left[\tfrac{1}{2} h\left(r^2 - h^2\right)^{1/2} + \tfrac{1}{2} r^2 \sin^{-1}\left(h/r\right)\right]$$

$$= h\left(r^2 - h^2\right)^{1/2} + r^2 \sin^{-1}\left(h/r\right)$$

$$Area\ of\ ABDA \quad = \tfrac{1}{2}\pi r^2 - A^1$$

$$= 0.5\pi r^2 - h\left(r^2 - h^2\right)^{1/2} - r^2 \sin^{-1}\left(h/r\right)$$

$$Volume \quad V \quad = L\left[0.5\pi r^2 - h\left(r^2 - h^2\right)^{1/2} - r^2 \sin^{-1}\left(h/r\right)\right]$$

$$f(h) \quad = V - L\left[0.5\pi r^2 - h\left(r^2 - h^2\right)^{1/2} - r^2 \sin^{-1}\left(h/r\right)\right]$$

given;

$$V = 12.4\ ,\ L = 10,\ r = 1$$

$$f(h) \quad = 12.4 - 10\left[0.5\pi - \sin^{-1}(h) - h\left(1 - h^2\right)^{1/2}\right]$$

given $\text{int}erval, (0.1,\ 0.18)$

$$f(0.1) = 12.4 - 10\left[0.5\pi - \sin^{-1}(0.1) - (0.1)\left(1 - (0.1)^2\right)^{1/2}\right]$$

$$= -1.311301619\ .\ (negetive)$$

$$f(0.18) = 12.4 - 10\left[0.5\pi - \sin^{-1}(0.18) - (0.18)\left(1 - (0.18)^2\right)^{1/2}\right]$$

$$= 0.272501142\ .\ (positive)$$

The root has within (0.1 , 0.18).

n = number of iterations.

a_n = negative value

b_n = positive value

$p_n = \dfrac{a_n + b_n}{2}$

Note:- for all calculations doing Radians mode in calculator.

n	a_n	b_n	p_n	$f(pn)$
1	0.1	0.18	0.14	-
2	0.14	0.18	0.16	-
3	0.16	0.18	0.17	+
4	0.16	0.17	0.165	-
5	0.165	0.17	0.1675	+

Meaning of "within 0.01 feet; when calculating root between two successive steps, first two decimal places become constant, then stop calculating and take final step as the root,

There fore root is: 0.1675

(04).

(a) $f(x) = x^3 + x^2 + x + 7$

So that $f(-2.2) = -1.008$ *and* $f(-2.1) = -0.049$

i.e. *a root between* (-2.2) *and* (-2.1)

$\therefore$ Taking $x_0 = (-2.2)$ *and* $x_1 = (-2.1)$

$f(x_0) = -1.008$, $f(x_1) = 0.049$

By the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= (-2.2) - \frac{0.1}{1.057}(-1.008)$$

$$= -2.104635762$$

Now $f(x_2) = f(-2.104635762) = 0.00238931312$

Taking $x_0 = -2.2$, $x_2 = -2.104635762$

$$x_3 = x_0 - \frac{x_2 - x_0}{f(x_2) - f(x_0)} f(x_0)$$

$$= (-2.2) - \frac{0.095364238}{1.010389313} \times (-1.008)$$

$x_3 = -2.104861274$

$f(x_3) = 0.0001606041$

Hence,

Root of the equation is -2.104861274 (Correct to three decimal places ;)

(b) $\cos x = 3x - 1$

$f(x) = 3x - 1\cos x = 0$

$f(0) = -2 \qquad f(\pi/4) = 0.64$

$\therefore f(x)$ has a root between $[0, \pi/4]$

$$x = \frac{1+\cos x}{3}$$

Let $x = g(x)$

Then $g(x) = \frac{1+\cos x}{3}$

$g'(x) = \frac{1}{3}(-\sin x)$

$$|g'(x)|_{max} = \left|\frac{-\sin x}{3}\right|_{max} = \left|\frac{-\sin(\pi/4)}{3}\right| = 0.2357 < 1$$

$\therefore g(x)$ has unique fixed point $\left[0, \pi/4\right]$

$\therefore$ Suppose $P_{n+1} = g(p_n)$

$$P_{n+1} = \frac{1+\cos p_n}{3}$$

n	p_n	$g(p_n)$
0	0.5	0.625860854
1	0.625860854	0.603486378
2	0.603486378	0.607787348
3	0.607787348	0.606971188
4	0.606971188	0.607126454
5	0.607126454	0.60709693
6	0.60709693	0.6071102545
7	0.6071102545	0.607101477

root x is $= 0.6071$ (Correct four significant figures).

(c). $f(x) = x^3 - 3x^2 + 7x - 8$

$f'(x) = 3x^2 - 6x + 7$

$f(2) = 2 \quad , \quad f(1) = -3$

Take initial value $x_0 = 1.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{\left(x_n^3 - 3x_n^2 + 7x_n - 8\right)}{\left[3x_n^2 - 6x_n + 7\right]}$$

n	x_n	x_{n+1}
0	1.5	1.684210526
1	1.684210526	1.673635652
2	1.673635652	1.673593059

$\therefore$ root $x = 1.673593059$ (Correct three decimal places).

(05).

(a)
$$\begin{aligned} 3x + 4y - z &= 8 \\ -2x + y + z &= 3 \\ x + 2y - z &= 2 \end{aligned}$$

By using gauss elimination method,

$$A = \begin{bmatrix} 3 & 4 & -1 \\ -2 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$Ax = B$$

$$\begin{bmatrix} 3 & 4 & -1 \\ -2 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix}$$

Consider the augmented matrix is $[A|B]$

$$\left[\begin{array}{ccc:c} 3 & 4 & -1 & 8 \\ -2 & 1 & 1 & 3 \\ 1 & 2 & -1 & 2 \end{array}\right] \longrightarrow (2)$$

Now, equation (2) is to be reduced to an upper triangular matrix;

$$\left[\begin{array}{ccc:c} 3 & 4 & -1 & 8 \\ -2 & 1 & 1 & 3 \\ 1 & 2 & -1 & 2 \end{array}\right] \xrightarrow{R_2 \to R_2 + \frac{2}{3}R1} \left[\begin{array}{ccc:c} 3 & 4 & -1 & 8 \\ 0 & 11/3 & 1/3 & 25/3 \\ 1 & 2 & -1 & 2 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_1/3}$$

$$\left[\begin{array}{ccc:c} 3 & 4 & -1 & 8 \\ 0 & 11/3 & 1/3 & 25/3 \\ 0 & 2/3 & -2/3 & -2/3 \end{array}\right] \xrightarrow{R_3 \to R_3 - \frac{2}{11}R_2} \left[\begin{array}{ccc:c} 3 & 4 & -1 & 8 \\ 0 & 11/3 & 1/3 & 25/3 \\ 0 & 0 & -24/33 & -72/33 \end{array}\right]$$

$$\begin{pmatrix} 3 & 4 & -1 \\ 0 & 11/3 & 1/3 \\ 0 & 0 & -24/33 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 25/3 \\ -72/33 \end{pmatrix}$$

$$AX = B$$

$-24/33\, z = -72/33$

$z = \frac{72}{24} = 3$

$\underline{z = 3}$

when $z = 3$;

$11/3\, y + 1/3\, z = 25/3$

$11/3\, y + 1/3 \times 3 = 25/3$

$11y = 22$

$\underline{y = 2}$

when $z = 3, \quad y = 2$

$3x + 4y - 2 = 8$

$3x + 8 - 3 = 8$

$\underline{x = 1}$

(b)

$$\begin{aligned} 2x - 3y + z &= -1 \\ x + 4y + 5z &= 25 \\ 3x - 4y + z &= 2 \end{aligned}$$

Write the given equations in the from,

$$x = \frac{1}{2}(-1 + 3y - z)$$

$$y = \frac{1}{4}(25 - x - 5z) \longrightarrow (1)$$

$$z = \frac{1}{1}(2 - 3x + 4y)$$

Approximate, $x_0 = 0, \quad y_0 = 0, \quad z_0 = 0,$

Substituting these on the right sides of these equations (1),

$$x_1 = -\tfrac{1}{2} \qquad x_1 = -0.5$$

$$y_1 = \frac{1}{4}(25) \qquad y_1 = -6.25$$

$$z_1 = 2 \qquad z_1 = 2$$

Putting those values, on the right of the equation (1),

$$x_2 = \tfrac{1}{2}(-1+3\times 6.25-2) \qquad x_2 = 7.875$$

$$y_2 = \frac{1}{4}(25-(-0.5)-5\times 2) \qquad y_2 = 3.875$$

$$z_2 = (2-3\times(-0.5)+4(6.25)) \qquad z_2 = 28.5$$

Substituting these on the right sides of these equations (1),

$$x_3 = \tfrac{1}{2}(-1+3\times 3.875-28.5) \qquad x_3 = -8.9375$$

$$y_3 = \frac{1}{4}(25-7.875-5\times 28.5) \qquad y_3 = -31\cdot 3437575$$

$$z_3 = (2-3\times(7.875)+4\times 3.875) \qquad z_3 = -6.125$$

Substituting those values we get.

$$x_4 = \tfrac{1}{2}(-1+3\times -31.34375-(-6.125)) \qquad x_4 = -44.453125$$

$$y_4 = \frac{1}{4}(25-(-8.9375)+5(31.34375)) \qquad y_4 = -30.6953125$$

$$z_4 = (2-3(-8.9375)+4(31.34375)) \qquad z_4 = -96.5625$$

Note:- The values in the two iterations being particularly same, we can stop.
(In this question diagonal are not large as compared to other coefficients there for difficult to received correct answer.)

(c)

$$3x+y+2z = 16$$
$$2x-6y+8z = 25$$
$$5x+4y-3z = 2$$

Write the given equations in the from,

$$x = \frac{1}{3}(16-y-2z)$$

$$y = \frac{-1}{6}(24-2x-8z) \longrightarrow (1)$$

$$z = \frac{-1}{3}(2-4y-5x)$$

Start approximation, $x_0 = 0, y_0 = 0, z_0 = 0$

Substituting these on the right sides of this equation (1), we get.

$x_1 = 5.333333333$

Putting $x = x_1, z = z_0$, in second equation of (i),

We obtain, $y_1 = -2.222222222$

Putting $x = x_1, y = y_1$, in equation (i),

$z_1 = 5.259259259$

For the second iteration,

$x_2 = 2.567901235$

$y_2 = 3.868312757$

$z_2 = 8.770919067$

For the third iteration,

$x_3 = -1.803383631, \quad y_3 = 7.09343088, \quad z_3 = 5.785601788$

$x_4 = -0.888211485, \quad y_4 = 3.418065223, \quad z_4 = 2.410401155$

$x_5 = 2.587044156, \quad y_5 = 0.076216258, \quad z_5 = 3.746695271$

Note:- In this equation diagonal are not large as compared to other coefficients. There for difficult to received correct answer

X	Y	Z
2.810131067	1.93230405	6.593290511
0.293704975	4.888955674	6.541449191
-0.523951352	4.280615138	4.16756793
1.128083001	1.932784907	3.790518211
2.162059557	1.7747108	5.303046995
1.206398403	3.472862128	5.974480176
0.192725839	4.030215515	5.028163753
0.637818993	2.916824668	4.285464546
1.50408208	2.215313421	4.793888028
1.398970174	2.858174096	5.475849084
0.730042578	3.544479639	5.276043816
0.634477576	3.246217613	4.719086112
1.105203388	2.660515945	4.722693573
1.298032303	2.729602198	5.136190103
0.999339198	3.181366537	5.240720713

(06).

(i) Newton forward interpolation formula,

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)\Delta^2 y_0}{2!} + \frac{p(p-1)(p-2)\Delta^3 y_0}{3!} + \ldots\ldots + \frac{+\,p(p-1)\quad[p-(n-1)]\Delta^n y_0}{n!} \text{ where } p = \frac{x-x_0}{h}$$

h is time interval

Newton backward interpolation formula,

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)\nabla^2 y_n}{2!} + \frac{p(p+1)(p+2)\nabla^3 y_n}{3!} + \ldots\ldots + \frac{+\,p(p+1)(p+2)\ldots\ldots[p+(n-1)]\nabla^n y_n}{n!}, \text{ where } p = \frac{x-x_n}{h}$$

h is time interval,

x	0.1	0.15	0.2	0.25	0.30
y	0.1003	0.1511	0.2027	0.2553	0.3093

Newton's interpolation table,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	0.1003				
		0.0508			
0.15	0.1511		0.0008		
		0.0516		0.0002	
0.2	0.2027		0.001		0.0002
		0.0526		0.0004	
0.25	0.2553		0.0014		
		0.054			
0.30	0.3093				

(a) Using Newton's forward interpolation formula,

$$\tan(0.12)$$

$$p = \frac{(x-x_0)}{h}$$

$$p = \frac{0.12-0.1}{(0.15-0.1)}$$

$$p = 0.4$$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)\Delta^2 y}{2!} + \frac{p(p-1)(p-2)\Delta^3 y_0}{3!} + \frac{p(p-1)(p-2)(p-3)\Delta^4 y_0}{4!}$$

$$\tan(0.12) = 0.1003 + (0.4)(0.0508) + \frac{0.4 \times -0.6 \times 0.0008}{2!}$$
$$+ \frac{0.4 \times (-0.6) \times (-1.6) \times (0.0002)}{3!} + \frac{(0.4 \times -0.6 \times -1.6 \times -2.6 \times 0.0002)}{4!}$$

$$\underline{\underline{\tan(0.12) = 0.12052848}}$$

(b) Using Newton's backward interpolation formula,

$$= \tan(0.26)$$

$$p = \frac{0.26 - 0.3}{0.5} = (-0.8)$$

$$y_p = y_n + p\Delta y_n + \frac{p(p+1)\Delta^2 y}{2!} + \frac{p(p+1)(p+2)\Delta^3 y}{3!} + \frac{p(p+1)(p+2)(p+3)\Delta^4}{4!}$$

$$= 0.3093 + (-0.8) \times 0.054 + \frac{(-0.8 \times 0.2 \times 0.0014)}{2!} + \frac{(-0.8 \times 0.2 \times 1.2 \times 0.000}{3!}$$
$$+ \frac{(-0.8 \times 0.2 \times 1.2 \times 2.2 \times 0.0002)}{4!}$$

$$\underline{\underline{\tan(0.26) = 0.26597168}}$$

(ii)

Age (less than x)	25	30	40	50
%of criminals	52	67.3	84.1	94.4

$$y = f(35)$$

Lagrange's interpolation formula,

$$F(x) = \frac{y_0(x - x_1) \quad (x - x_n)}{(x_0 - x_1)(x_0 - x_1)...(x_0 - x_n)} f(x_0) + \frac{\overline{y}_1(x - x_1)....(x - x_n) f(x_1)}{(x - x_0)(x_1 - x_2)...(x_1 - x_n)} +$$

$$.....+ \frac{y_n(x - x_0)(x - x_1).....(x - x_{n-1})}{(x_n - x_0)(x_n - x_0)...(x_n - x_{n-1})} f(x_0)$$

Apply above formula,

$$= \frac{52(35-30)(35-40)(35-50)}{(25-30)(25-40)(25-50)} + \frac{67.3(35-25)(35-40)(35-50)}{(30-25)(30-40)(30-50)}$$

$$+\frac{84.1(35-25)(35-30)(35-50)}{(40-25)(40-30)(40-50)}+\frac{94.4(35-25)(35-30)(35-40)}{(50-25)(50-30)(50-40)}$$

$$=\frac{52\times5\times(-5)\times(-15)}{(-5)(-15)\times(-25)}+\frac{67.3\times10\times-5\times-15}{5\times-10\times-20}+\frac{84.1\times10\times5\times-15}{15\times10\times-10}$$

$$+\frac{94.4\times10\times5\times-5}{25\times20\times10}$$

$$\underline{\underline{f(35) = 77.405}}$$