The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Continuous Assessment Test (NBT) - 2017/2018
Pure Mathematics - Level 04
PEU4300/PEE4300– Real Analysis I

Duration: - One Hour

Date: - 30-07-2018 **Time: - 4:15p.m – 5.15p.m.**

Answer **all** the questions.

1. Let $S_n = \sum_{r=1}^{n}(-1)^r$. Find S_{100} and S_{2017} . (10 marks)

2. Is $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{2n-3}$ a geometric series? If it is, find the initial term and the common ratio of it. If it is not, explain your answer. (10 marks)

3. Prove that the series $\sum_{n=1}^{\infty}\frac{1}{(n)(n+2)}$ converges and find its sum. (15 marks)

4. Prove that $\sum_{r=N+1}^{2N}\frac{1}{r} \geq \frac{1}{2}$, where $N \in \mathbb{N}$. (10 marks)

5. Find two positive divergent series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, such that $\sum_{n=1}^{\infty} a_n b_n$ convergent. (10 marks)

6. Let $\langle u_n \rangle$ be a positive sequence. Prove that, if $\sum_{n=1}^{\infty} u_n$ convergent then $\sum_{n=1}^{\infty}\frac{1}{u_n}$ is divergent. (10 marks)

7. Find whether each of the following series are convergent or divergent. Justify your answer.

(i) $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$ (ii) $\sum_{n=1}^{\infty}\frac{\left(\sqrt{n+1}-\sqrt{n}\right)}{n}$ (iii) $\sum_{n=1}^{\infty}\frac{1}{(2^n+(-2)^n+2)}$

(iv) $\sum_{n=1}^{\infty}\frac{(7-3n)}{(3n^2-5)}$ (v) $\sum_{n=1}^{\infty}\left(\frac{n^n}{3^{25n}}\right)$ (25 marks)

8. Find the radius of convergence of $\sum_{n=1}^{\infty}\left(\frac{2^n}{3^n}\right)x^n$ and $\sum_{n=1}^{\infty}\left(\frac{1}{n^3 3^n}\right)x^n$. (10 marks)