THE OPEN UNIVERSITY OF SRI LANAKA
DIPLOMA IN TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2010/2011
MPZ 3230 – ENGINERING MATHEMATICS I
DURATION: THREE (03) HOURS

Date: 05th March 2011 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer only six (06) questions, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 06.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

Important integrals

- $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
- $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
- $\int f(x)e^{ax}dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} - .....$

  Sign alternate $(+-+-+-.....)$
- $\int f(x)\cos axdx = \frac{1}{a}f(x)\sin ax + \frac{1}{a^2}f^1(x)\cos ax - \frac{1}{a^3}f^2(x)\sin ax.....$

  Sign alternate in pairs $(++--++--++--.....$
- $\int f(x)\sin axdx = -\frac{1}{a}f(x)\cos ax + \frac{1}{a^2}f^1(x)\sin ax + \frac{1}{a^3}f^2(x)\cos ax.....$

  Sign alternate in pairs after the first term $(++--++--++--.....$
- $\int \sec axdx = \frac{1}{a}\ln|\sec ax + \tan ax|$

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SECTION - A

1. Solve the following differential equations

   (a) $\frac{dy}{dx}+3\alpha y=x^2$

   (b) $\frac{dy}{dx}=\frac{4x+3y}{3x+4y}$

   (c) $(x+3)\frac{dy}{dx}=(x^2+4x+5)$

2. (a) Using a trial function find a particular solution of the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=34e^{3x}$. Hence find the general solution of the above differential equation.

   (b) Using the formula $\frac{1}{D+\alpha}f(x)=e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$ find the particular integral of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=5sin3x$. Hence find the general solution of the above differential equation

3. (a) Find the Laplace transformation of $f(t)=\begin{cases}2 & , \ 0<t\leq 5\\ t^3 & , \ 5\leq t\end{cases}$

   (b) Using Laplace transformation find the solution of the following differential equation $\frac{d^2y}{dx^2}+y=3sin2x$, where $x\geq 0$, $y(0)=1$ and $\left(\frac{dy}{dx}\right)_{x=0}=2$

4. (a) Find the factors of the following determinants by using row and column operations.

   (i) $\begin{vmatrix} a & b & c & d\\ b & c & d & a\\ c & d & a & b\\ d & a & b & c\end{vmatrix}$ (ii) $\begin{vmatrix} 1 & a^2 & a^3\\ 1 & b^2 & b^3\\ 1 & c^2 & c^3\end{vmatrix}$.

   (b) Show that $\begin{bmatrix} cos\theta & 0 & sin\theta\\ 0 & 1 & 0\\ -sin\theta & 0 & cos\theta\end{bmatrix}$ is an orthogonal matrix.

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5. Using the augmented matrix and elementary row operations discuss the solutions of the following systems of equations

(a)

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (\mu^2 - 14)z = \mu + 2 \quad \text{Where } \mu \in R$$

(b)

$$x + y = 2$$
$$x - y = 0$$
$$3x + y = t, \text{ where } t \in R.$$

6. (a) Define the Taylor polynomial of degree n.

If $f(x) = \ln(cosx)$ prove that $\frac{d^3f(x)}{dx^3} + 2\frac{d^2f(x)}{dx^2}\frac{df(x)}{dx} = 0$.

Hence find Taylor polynomial of degree four of $f(x) = \ln(cosx)$ about $x = 0$ .Deduce that $ln2 \approx \frac{\pi^2}{16}\left(1 + \frac{\pi^2}{96}\right)$

(b) Define the Taylor series expansion of $f(x)$ about $x = a$.

Prove that Taylor series expansion of $\frac{1}{x}$ about $x = 3$ is $\sum_{r=0}^{\infty} \frac{(-1)^r(x-3)^r}{3^{r+1}}$.

Show also that the above series converges to $\frac{1}{x}$ and find the radius of convergence.

(Hint: Use the convergence of a geometric series.)

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SECTION – B

7 Define the scalar (dot) and vector product of two vectors.

(a) The position vectors of the points A, B, C are given by $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$

(i) Find the value of λ such that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(ii) Find the area of the triangle ABC.

(b) Examine whether the vectors $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ are linearly independent or dependent.

(c) Find a unit vector perpendicular to both vectors $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + \mathbf{k}$.

8. (a) If $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are the position vector, velocity and acceleration of the particle at time t respectively. Prove that $\frac{d}{dt}(\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})) = 2(\boldsymbol{v} \cdot \boldsymbol{a})\boldsymbol{r} - (\boldsymbol{r} \cdot \boldsymbol{a})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{r})\boldsymbol{a}$.

(b) Prove that if $f(t) = \frac{e^t sint}{\sqrt{1+e^{2t}}}\mathbf{i} + \frac{e^t cost}{\sqrt{1+e^{2t}}}\mathbf{j} + \frac{1}{\sqrt{1+e^{2t}}}\mathbf{k}$

(i) $|f(t)| = 1$

(ii) $|f(t)|$ is perpendicular to $\frac{d}{dt}(f(t))$

(iii). $\boldsymbol{r} \cdot \frac{d^2(f(t))}{dt^2} = -\left|\frac{d(f(t))}{dt}\right|^2$

9. (a) Using a suitable interpolation method and the following table, find $f(15)$

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

(b) Show that a real root of the equation $xe^x - 2 = 0$ lies between 0 and 1. Using Newton Raphson method, find a real root of the equation $xe^x - 2 = 0$, correct to two decimal places.

(c) Using Lagrange's interpolation formula, find the polynomial passing through the points $(1,-7),(3,5),(4,8),(6,14)$.

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10. (a) The events A and B are such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$ and $P(A \backslash B) = \frac{1}{5}$.

Find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A' \backslash B)$ (iv) $P(A \backslash B')$

State whether A and B are mutually exclusive or independent .Justify your answer.

(b) A student attempts a multiple choice question to which 4 possible answers are given with only one of them being correct. For any question there is a probability of $\frac{2}{3}$ that he knows the correct answer. If he does not know the correct answer, then he will mark the answer at random. He does, in fact, mark the correct answer. What is the probability that he knew the correct answer?

11. (a) The probability density function of a discreet random variable X is

$P(X = x) = k\left(\frac{3}{4}\right)^x$ Where $x = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$

(i) Find the value of constant k.

(ii) Find $P(X > 8)$ and $P(5 \leq X < 20)$

(b) The probability density function of a discreet random variable is given by

x	1	2	3	4	5
$P(X = x)$	$0 \cdot 1$	$0 \cdot 3$	$0 \cdot 2$	$0 \cdot 3$	$0 \cdot 1$

(i) Find $E(X)$ (ii) $V(X)$ (iii) $E(4X^2 + 3X - 1)$

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12. The time taken to fix a certain electronic item, in t hours, has the probability density function $f(t) = \begin{cases} 10kt^2 & , \ 0 \le t < 0.6 \\ 9k(1-t) & , \ 0.6 \le t \le 1.0 \\ 0 & , \ \textit{otherwise} \end{cases}$, where k is a constant.

a) Find the value of k

b) Sketch the graph of $f(t)$.

c) Write down the most likely time (mode).

d) Find the expected time.

e) Determine the probability that the time will be

(i) more than 48 minutes, (ii) between 24 and 48 minutes.

END

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