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THE OPEN UNIVERSITY OF SRI LANAKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2010/2011

MPZ 3132 – ENGINERING MATHEMATICS IB

DURATION: THREE (03) HOURS

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Date: 05th March 2011 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer only five (05) questions, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 07.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**
- **Last page contains the table of Normal Distribution**

Important integrals

- $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
- $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
- $\int f(x)e^{ax}dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} - .....$

  Sign alternate $(+--+--+-.....)$
- $\int f(x)\cos axdx = \frac{1}{a}f(x)\sin ax + \frac{1}{a^2}f^1(x)\cos ax - \frac{1}{a^3}f^2(x)\sin ax.....$

  Sign alternate in pairs $(++--++--++--.....$
- $\int f(x)\sin axdx = -\frac{1}{a}f(x)\cos ax + \frac{1}{a^2}f^1(x)\sin ax + \frac{1}{a^3}f^2(x)\cos ax.....$

  Sign alternate in pairs after the first term $(++--++--++--.....$
- $\int \sec axdx = \frac{1}{a}\ln|\sec ax + \tan ax|$

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SECTION – A

1. The function f is defined as $f(x) = x^2 \quad x \in (0, 1)$

   a) Extend the above function as a

      (i) Odd periodic function with period 2

      (ii) Even periodic function with period 2

      (iii)Periodic function with period 1

   Marks 40

   b) Draw the graphs of the above three functions on $[-3, 3]$

   Marks 30

   c) Find the Fourier representation of $f(x) = x^2 \quad x \in (0, 1)$

      (i) As a sine series with period 2

      (ii) As a cosine series with period 2

      (iii)As a full trignometric series with period 1

   Marks 130

2. (a) Define the Taylor series expansion of $f(x)$ about $x = a$.

   Prove that Taylor series expansion of $\frac{1}{x}$ about $x = 3$ is $\sum_{r=0}^{\infty} \frac{(-1)^r (x-3)^r}{3^{r+1}}$.

   Show also that the above series converges to $\frac{1}{x}$ and find the radius of convergence.

   (Hint: Use the convergence of a geometric series.)

   Marks 70

   (b) If $\boldsymbol{r}, \boldsymbol{v}$ and $\boldsymbol{a}$ are the position vector, velocity and acceleration of a particle at time t respectively.

   Prove that $\frac{d}{dt}(\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})) = 2(\boldsymbol{v} \cdot \boldsymbol{a})\boldsymbol{r} - (\boldsymbol{r} \cdot \boldsymbol{a})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{r})\boldsymbol{a}.$

   Marks 50

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(c) Prove that if $f(t) = \frac{e^t sint}{\sqrt{1+e^{2t}}}\mathbf{i} + \frac{e^t cost}{\sqrt{1+e^{2t}}}\mathbf{j} + \frac{1}{\sqrt{1+e^{2t}}}\mathbf{k}$

(i) $|f(t)| = 1$

(ii) $|f(t)|$ is perpendicular to $\frac{d}{dt}(f(t))$

(iii). $f(t) \cdot \frac{d^2(f(t))}{dt^2} = -\left|\frac{d(f(t))}{dt}\right|^2$

Marks 80

3. (a) The times taken to germinate for a certain kind of paddy seeds are normally distributed. If 20% of the seeds take more than six days to germinate and if 10% of the seeds germinate in less than 4 days, find the mean and the standard deviation of the time for germination of the paddy seeds.

Marks 140

(b) The height of a large batch of a certain crop are normally distributed with a mean of 140 cm and a standard deviation of 6cm.

(i) Find the probability that the height of a randomly selected tree of that batch is less than 145 cm .

(ii) If five trees are selected at random from the above batch ,find the probability that all the five trees are less than 145 cm tall and only three of them are less than 145 cm .

Marks 60

4. (a) A particle of mass m is projected vertically upwards with velocity u in a resistive medium whose resistance is mkv^2 ,where v is the speed of the particle and k is a constant. Prove that the greatest height reached by the particle is $\frac{1}{2k} ln\left(1+\frac{ku^2}{g}\right)$

Marks 60

(b)A uniform rod of mass m and length 2a can turn freely about one end which is fixed. It is given an angular velocity ω from the position in which it hangs vertically. Show that if θ is the inclination of the rod to the down-ward vertical at time t, then

$\dot{\theta}^2 = \omega^2 - \frac{3g}{2a}(1 - cos\theta)$.

Find the least value ω_0 of ω so that the rod may just make complete revolutions. With this particular angular velocity ω_0, show that the time of describing any angle $\theta(< \pi)$ is

$2\sqrt{\frac{a}{3g}} ln\left[tan\left(\frac{\pi+\theta}{4}\right)\right]$.

Marks 140

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SECTION – B

5. (a) Using a method of trial function find a particular solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 34e^{3x}$. Hence find the general solution of the above differential equation .

Marks 65

(b) Prove that $\frac{1}{D+\alpha} f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x} f(x)$.

Using the above formula find the particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5sin3x$ .Hence find the general solution of the above differential equation.

Marks 135

6. (a) Shade the region satisfying the inequality $Re(z+2) \geq |z-2|$ and $|z| \leq 2$.

Marks 70

(b) Prove that $Log(1+i) = \frac{1}{2}ln2 + i(2k + \frac{1}{4})\pi$ where $k \in Z$.

Marks 45

(c) If $f(z) = \frac{4z+3}{z-1}$ where $z \in C\backslash\{1\}$, then show that f is a one to one function and find the inverse function of f.

Marks 25

(d) If the function g is defined as $g(z) = z^2$, $|z| \leq 2$ and $Im(z) \geq 0$,then find the image of the function and shade the domain and the range of g.

Marks 60

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7. (a) Using the augmented matrix and elementary row operations ,discuss the solution of the following system of equations .When the system has solutions ,find those solutions.

$$x + 2y - 3z = 4.$$

$$3x - y + 5z = 2.$$

$$4x + y + (\mu^2 - 14)z = \mu + 2 \quad \text{Where } \mu \in R$$

Marks 125

(b) Find the solution of the following differential equation using Laplace transformation.

$\frac{d^2y}{dx^2} + y = 3sin2x$,where $x \geq 0$, $y(0) = 1$ and $\left(\frac{dy}{dx}\right)_{x=0} = 2.$

Marks 75

8. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance

$$\frac{2}{3}\left(\frac{h_1^2 + h_1h_2 + h_2^2}{h_1 + h_2}\right)$$

from the surface.

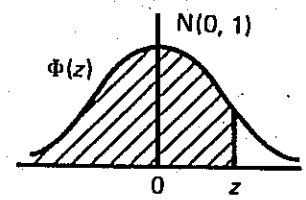
Marks 120

The inside cross section of a closed right cone is a square of side $2a$ and the height of the inside of the cone is h. The cone is completely filled with a homogeneous liquid. The weight of the liquid inside the cone is w . If the cone is kept such that the axis of the cone is horizontal, find the reaction on the angled surfaces of the cone.

Marks 80

END

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THE DISTRIBUTION FUNCTION $\Phi(z)$ OF THE NORMAL DISTRIBUTION N(0, 1)

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											ADD								
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	36
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	7	11	15	19	22	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	20	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	7	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	5	8	11	14	16	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	19	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	18
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	17
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	4	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898								0	1	1	1	1	2	2	2	2
				.9901	.99036	.99061	.99086				3	5	8	10	13	15	18	20	23
								.99111	.99134	.99158	2	5	7	9	12	14	16	18	21
2.4	.99180	.99202	.99224	.99245	.99266						2	4	6	8	11	13	15	17	19
						.99286	.99305	.99324	.99343	.99361	2	4	6	7	9	11	13	15	17
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520	2	3	5	6	8	9	11	12	14
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643	1	2	3	5	6	7	8	9	10
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736	1	2	3	4	5	6	7	8	9
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807	1	1	2	3	4	4	5	6	6
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861	0	1	1	2	2	3	3	4	4
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900	0	1	1	2	2	2	3	3	4
3.1	$.9^3032$	$.9^3065$	$.9^3096$								3	6	9	13	16	19	22	25	28
				$.9^3126$	$.9^3155$	$.9^3184$	$.9^3211$				3	6	8	11	14	17	20	22	25
								$.9^3238$	$.9^3264$	$.9^3289$	2	5	7	10	12	15	17	20	22
3.2	$.9^3313$	$.9^3336$	$.9^3359$	$.9^3381$	$.9^3402$						2	4	7	9	11	13	15	18	20
						$.9^3423$	$.9^3443$	$.9^3462$	$.9^3481$	$.9^3499$	2	4	6	8	9	11	13	15	17
3.3	$.9^3517$	$.9^3534$	$.9^3550$	$.9^3566$	$.9^3581$						2	3	5	6	8	10	11	13	14
						$.9^3596$	$.9^3610$	$.9^3624$	$.9^3638$	$.9^3651$	1	3	4	5	7	8	9	10	12
3.4	$.9^3663$	$.9^3675$	$.9^3687$	$.9^3698$	$.9^3709$	$.9^3720$	$.9^3730$	$.9^3740$	$.9^3749$	$.9^3758$	1	2	3	4	5	6	7	8	9
3.5	$.9^3767$	$.9^3776$	$.9^3784$	$.9^3792$	$.9^3800$	$.9^3807$	$.9^3815$	$.9^3822$	$.9^3828$	$.9^3835$	1	1	2	3	4	4	5	6	7
3.6	$.9^3841$	$.9^3847$	$.9^3853$	$.9^3858$	$.9^3864$	$.9^3869$	$.9^3874$	$.9^3879$	$.9^3883$	$.9^3888$	0	1	1	2	2	3	3	4	5
3.7	$.9^3892$	$.9^3896$	$.9^390$	$.9^404$	$.9^408$	$.9^412$	$.9^415$	$.9^418$	$.9^422$	$.9^4250$									
3.8	$.9^428$	$.9^431$	$.9^433$	$.9^436$	$.9^438$	$.9^441$	$.9^443$	$.9^446$	$.9^448$	$.9^4500$									
3.9	$.9^452$	$.9^454$	$.9^456$	$.9^458$	$.9^459$	$.9^461$	$.9^463$	$.9^464$	$.9^466$	$.9^4670$									

For negative values of z use $\Phi(z) = 1 - \Phi(-z)$