The Open University of Sri Lanka
Department of Mathematics
B.Sc/ B.Ed Degree Programme
No Book Test - 2017/ 2018
Pure Mathematics– Level 05
PEU5302– Combinatorics

DURATION: ONE HOUR

Date: 16 – 07 – 2018 **Time: 4.15 p.m. – 5.15 p.m.**

ANSWER ALL QUESTIONS.

Let n and r be two positive integers such that $1 \le r \le n$ and let ${}^nC_r$ be the coefficient of the $(r+1)^{\text{th}}$ term of the binomial expansion of $(1+x)^n$.

01. Determine the sum of the coefficients of the terms with even powers of x in the multinomial expansion of $(1+x-x^2)^n$. [15 mark]

02. Prove the Pasçal's identity by using the combinatorial arguments. [10 mark]

Hence, deduce that ${}^{n+2}C_{r+2} = {}^nC_{r+2} + 2\,{}^nC_{r+1} + {}^nC_r$. [10 mark]

03. Prove each of the following statements using ONLY the combinatorial arguments:

(i) ${}^{3n}C_2 - {}^{2n}C_2 - {}^nC_2 = 2n^2$, [15 mark]

(ii) $n \times {}^{n-1}C_{r-1} = r \times {}^nC_r$. [15 mark]

04. Find the coefficient of $x^5 y^4 z^3 w^3$ in the expansion of $(x+y+z+w)^{15}$. [10 mark]

05. Show each of the followings, when n is odd,

(i) ${}^nC_0 - {}^nC_1 + {}^nC_2 + \ldots - {}^nC_n = 0$, [10 mark]

(ii) $\left({}^nC_0\right)^2 - \left({}^nC_1\right)^2 + \left({}^nC_2\right)^2 + \ldots - \left({}^nC_n\right)^2 = 0$. [15 mark]