The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Continuous Assessment Test (OBT) - 2017/2018
Pure Mathematics - Level 04
PEU4300/PEE4300– Real Analysis I

Duration: - One Hour

Date: - 21-06-2018 **Time: - 4:15p.m – 5.15p.m.**

Answer **all** the questions.

1. Let $a, b, c, d \in \mathbb{R}$ such that $< b < c < d$. Prove that $\frac{a+b+c}{3} < \frac{a+b+c+d}{4}$. (10 marks)

2. Let $\langle x_n \rangle$ be the sequence given by $x_n = \frac{n-10}{n+1}$. Prove that $\langle x_n \rangle$ is an increasing sequence. Also prove that $\langle x_n \rangle$ is a bounded sequence. (15 marks)

3. Use the ε-definition of limit to prove that $\lim_{n\to\infty} \frac{1-2n}{2n+1} = -1$ (15 marks)

4. Find two sequences $\langle a_n \rangle$, $\langle b_n \rangle$ such that $\{a_n | n \in \mathbb{N}\}$ is infinite, $\{a_n | n \in \mathbb{N}\} = \{b_n | n \in \mathbb{N}\}$, and $a_n \neq b_n$ for all $n \in \mathbb{N}$. (15 marks)

5. Can you find $m \in \mathbb{N}$ such that $\left|\frac{3m+1}{2m+1} - \frac{1}{2}\right| < \frac{4}{5}$? Justify your answer. (15 marks)

6. Use the definition of a Cauchy sequence to prove that the sequence $\langle 3 + (-1)^n \rangle$ is not a Cauchy sequence. (15 marks)

7. (a) Write down a sequence $\langle a_n \rangle$ of real numbers which diverges to$+\infty$,
(b) Write down a sequence $\langle b_n \rangle$ of real numbers which diverges to $-\infty$
(c) Write down a sequence $\langle c_n \rangle$ of real numbers such that which has three sub sequences the first diverges to$+\infty$, the second diverges to $-\infty$ and the third diverges to 5.
(15 marks)