00580

THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 04

FINAL EXAMINATION – 2013/2014

MPZ4230 – ENGINEERING MATHEMATICS II

DURATION: THREE (03) HOURS

Date: 19th August 2014 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer only six (6) questions.
- State any assumptions you required.
- This paper contains six (6) pages.
- Show all your workings.
- All symbols are in standard notation.

1. Given that the function, $f(x,y) = x^3 - 12x + y^3 + 3y^2 - 9y$.

   a). i. Write down all the first and second order partial derivatives of the function $f(x,y)$. [15%]

   ii. Find the maximum, minimum and saddle points of the function $f(x,y)$. [30%]

   b). Let $u = (x^2 + y^2 + z^2)^{1/2}$. Prove that

   i. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$

   ii. $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$ [30%]

   c). The frequency f on a LC circuit is given by

   $$f(x,y) = \frac{x^{-1/2}\, y^{-1/2}}{2\pi},$$

   where x is the inductance and y is the capacitance. If x is decreased by 1.5% and y is decreased by 0.5% then find the approximate percentage change in f. [25%]

2. a). Verify that whether the function, $f(z) = u + iv$ is analytic or not. [15%]

   b). Show that the function $u(x,y) = x^2 - y^2 - e^{2x}\cos 2y$ is a harmonic function. [15%]

   c). Find the function $v(x,y)$ such that $f(z) = u + iv$ is analytic,

   where $u(x,y) = x^2 - y^2 - e^{2x}\cos 2y$. [30%]

   d). Express $f(z)$ of part (c) in terms of z where $z = x + iy$. [10%]

   e). Compute the following integral:

$$I = \int_0^{\pi} \frac{d\theta}{2 - \cos\theta}$$

[30%]

3. a). Consider the simple linear regression model, $y = \beta_0 + \beta_1 x + \epsilon$.

   i. With usual notation, show that $\Sigma(y_i - \bar{y})^2 = \Sigma(y_i - \hat{y}_i)^2 + \Sigma(\hat{y}_i - \bar{y})^2$. [05%]

   ii. By using part (i) estimate RSS, TSS and ESS. [20%]

   b). A telecommunication company has been collect number of customers arrived at a particular time and going to know about the relationship between the number of customers and arrival time. They take a sample of 14 observations.

   i. Fill in the blanks of following tables of simple linear regression output by showing the way of calculation. [45%]

ANOVA

Model	Sum of Squares (SS)	Degrees of freedom (df)	Mean Square (MS)	F statistic
Regression	----------	----------	----------	----------
Residual	----------	----------	29.071	
Total	----------	----------	----------	

Coefficients

Model	Coefficient	Standard Error	t Statistic
Constant	4.162	----------	1.240
Units	----------	0.505	----------

$n =$ ---------- $\sigma^2 =$ ---------- $R^2 =$ ----------

ii. Write down the fitted regression line. [05%]

iii. Using the ANOVA table, test the hypothesis at **1%** significance level. [10%]

iv. Test the significance of the coefficients at **5%** significance level [10%]

v. Do you think that the fitted model describes the relationship between the two variables? Justify your answer. [05%]

4. a). Two batches of a certain chemical were delivered to a factory. For each batch ten determinates were made of the percentage of the manganese in the chemical. The results were as follows. Assume that the population variances are not equal.

Batch 1	3.3	3.7	3.5	4.0	3.4	3.5	4.1	3.8	3.2	3.7
Batch 2	3.5	3.6	3.2	3.4	3.0	3.4	2.8	3.1	3.3	3.6

i. Calculate the mean and standard deviation for each batch. [20%]

ii. Test whether there is a significant difference between the two batches. What would be your null and alternative hypothesis? [10%]

iii. Test the above hypothesis using **5%** significance level by considering the critical region and determine whether to accept or reject null hypothesis. [30%]

b) A company produces nails in batches of **10,000**. A guarantee for each batch should be given with **95%** confidence. For that a random sample of **50** nails were drawn randomly from each batch and checked for defectives. It was found that 6 defectives were found for the each batch considered. Using your observations, give the **95%** confidence interval for the proportion of defectives in the batch. [40%]

5. a) Using the 4^{th} order Taylor's series method, find $y(1.1)$ and $y(1.2)$ by solving following second order ordinary differential equation with boundary conditions : [50%]

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} - x^3 = 0, \qquad y(1) = 1 \text{ and } \left(\frac{dy}{dx}\right)_{x=1} = 1$$

b) i. State and prove Euler's method for solving an ordinary differential equation of the form $\frac{dy}{dx} = f(x, y)$. [10%]

ii. Use Euler's method to find a solution at $x = 1$ with step size $h = 0.25$.

$\frac{dy}{dx} = x + 2y;\ \ y(0) = 0.$ [20%]

iii. Let $\frac{dy}{dx} = x - y; y(0) = 0.5; x \in [0,1]$. Find $y(0.25)$ and $y(0.5)$ by using Runge-Kutta second order method. [20%]

6. a) Classify the elliptic, parabolic and hyperbolic form of the second order partial differential equation. [10%]

b) i. Suppose that a point O is at the center and A, B, P and Q are surrounding points such that $PO = QO = h$, $OA = \alpha h$ and $OB = \beta h$, $0 \leq \alpha \leq 1,\ 0 \leq \beta \leq 1$. If lines OP and OA are parallel to the X axis and OQ and OB are parallel to the Y axis.

Using Taylor Series expansion, obtain the expressions of $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$. [35%]

ii. Using part b (i), obtain the five point scheme for the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

[10%]

c) Apply the above scheme for the Laplace equation for the region with boundary values as shown in *Figure1*, and write down the equations in matrix form.

Hence find solutions at the points u_{11}, u_{12} and u_{21}

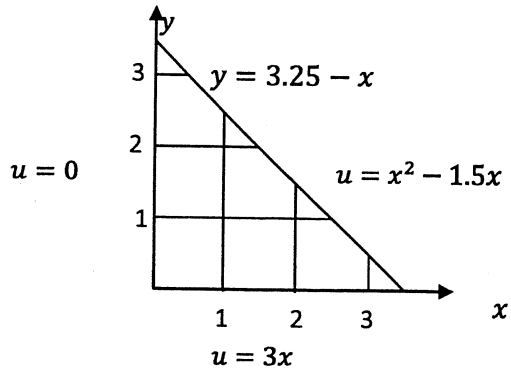


Figure 1

[45%]

7. In the series RLC circuit, inductance $(L) = 1$ Henry, resistance $(R) = 2$ ohms, capacitance $(C) = 0.1$ farad and voltage $(V_0) = 4\sin 3t$ volts.

   a) Find the capacitor charge at any time t. [80%]

   b) Using above result find current and capacitor voltage at time t. [20%]

Assume that,

Kirchhoff's voltage law for series circuit yield

$$V_L + V_R + V_C = V_0, \quad \text{where}$$

$$V_L = L\frac{di}{dt}, \quad V_R = iR, \quad V_C = \frac{1}{C}\int idt, \quad i = \frac{dq}{dt}$$

8. a) Given that the vectors $\underline{u_1} = (1,-1,1)$, $\underline{u_2} = (1,0,1)$ and $\underline{u_3} = (1,1,2)$ where $\underline{u_i} \in \mathbb{R}^3$, $i = 1, \ldots, 3$.

   i. Consider the $\mathbb{R}^3$ as a Euclidean inner product space. Find the orthogonal vectors of the above vectors using Gram Schmidt process. [30%]

   ii. Using above result verify the following identity for any two orthogonal vectors $\underline{u}$ and $\underline{v}$.

$$\|\underline{u} + \underline{v}\|^2 = \|\underline{u}\|^2 + \|\underline{v}\|^2$$ [10%]

   b) A linear transformation $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ is defined by,

$$T(M) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} M - M \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; M \in \mathbb{R}^{2\times 2}.$$

   i. Find the matrix $T(M)$ with respect to the standard basis of $\mathbb{R}^{2\times 2}$. [15%]

   ii. Find bases of the image and kernel of T. [30%]

   iii. Determine the rank and nullity of transformation T. [15%]

9. Let $A = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{pmatrix}$

a) Find the eigen values of A and its corresponding eigen vectors. [35%]

b) Find an orthogonal matrix P such that $D = P^{-1}AP$ where D is a diagonal matrix. [25%]

c) Using part (b) prove that, $D^{-1} = P^{-1}A^{-1}P$ and $D^n = P^{-1}A^nP$ [15%]

d) Derive the quadratic functions of $Q(\underline{x}) = \underline{x}^T A\underline{x}$ and $Q(\underline{y}) = \underline{y}^T D\underline{y}$

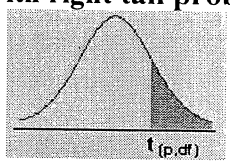
where $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ [10%]

e) Show that $P^{-1} = P^T$, and hence Show that $Q(\underline{x}) = Q(\underline{y})$ where $\underline{y} = P^T\underline{x}$ [15%]

-END-

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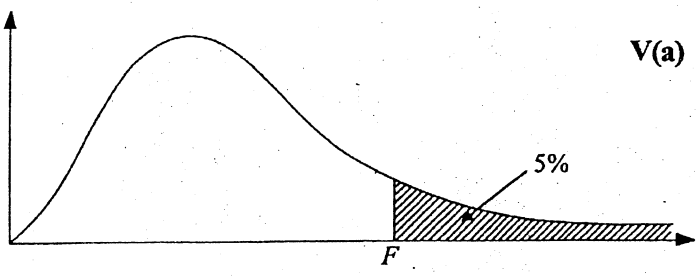
t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

10

V(a)



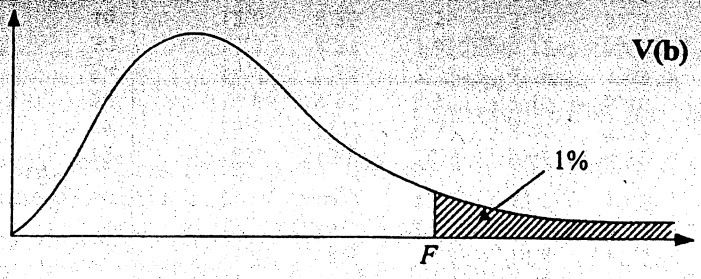
5 ප්‍රතිශතයට අනුරූප F ව්‍යාප්තිය

F ව්‍යාප්තියේ 5 ශතවීත ලක්ෂ්‍ය

5 percent points of the F distribution

n_2 \ n_1 =	1	2	3	4	5	6	7	8	10	12	24
2	18.5	19.0	19.20	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.29
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.08
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	1.98
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	1.89
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.08	2.00	1.79
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.70

V(b)



1 ප්‍රතිශතයට අනුරූප F ව්‍යාප්තිය

F ව්‍යාප්තියේ 1 ශතවීත ලක්ෂ්‍ය

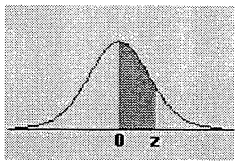
1 percent points of the F distribution

n_2 \ n_1 =	1	2	3	4	5	6	7	8	10	12	24
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.6
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	13.9
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.1	9.89	9.47
6	13.7	10.98	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.31
7	12.3	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.07
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.28
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.73
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.33
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	3.78
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.80	3.67	3.29
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.37	3.23	2.86
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.17	3.03	2.66
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.98	2.84	2.47
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.80	2.66	2.29
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.63	2.50	2.12

11

Standard Normal (Z) Table

Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Probabilities for the standard normal distribution

00580

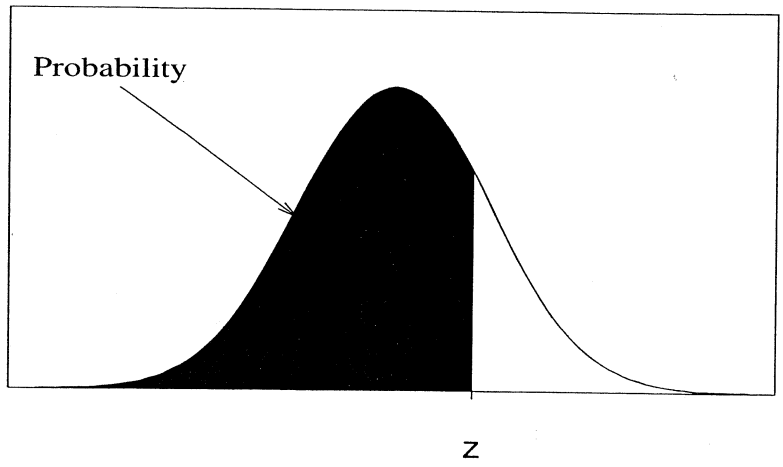


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998