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The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Diploma in Technology/Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX4272 Vibration and Fault Diagnosis**
Academic Year	:	2013/14
Date	:	07th September 2014
Time	:	1330 hrs. – 1630 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

QUESTION 01

Fig. Q1 shows a vibratory system in which a uniform rod of mass m_1 and length $2a$ is pivoted at its lower end. One end of a spring of stiffness k_1 is fixed to the mid point of the rod and the other end is fixed to a vertical support. One end of a string which runs over a smooth pulley is attached to the upper end of the rod while the other end carries a mass m_2 supported on a second spring of stiffness k_2. The rod is vertical in the equilibrium position.

(a) Determine the equation of motion of the system for small oscillations and the natural frequency of vibration.

(b) If, $m_1 = m_2 = m$, $k_1 = k$ and $k_2 = k/2$, show that the natural frequency (ω_n) can be expressed as

$$\omega_n = \frac{1}{4}\sqrt{9\left(\frac{k}{m}\right) - 3\left(\frac{g}{a}\right)}.$$

(c) If, $m = 1$ kg, $k = 200$ N/m and $a = 400$ mm, calculate the natural frequency.

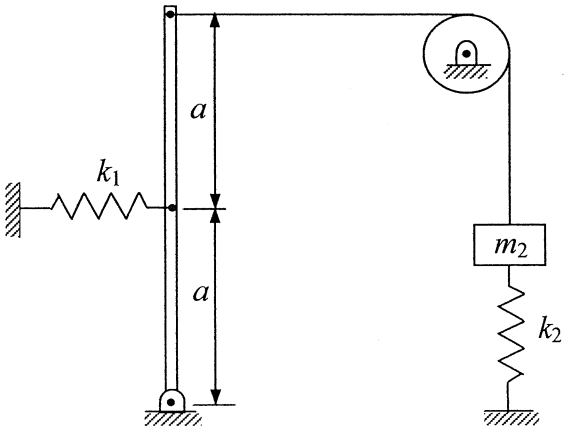


Fig. Q1

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QUESTION 02

Fig. Q2 shows a spring-mass-damper system in which the movement of the mass m is restrained by springs of stiffnesses k_1, k_2, k_3 and a damper of damping coefficient b.

(a) Determine the equation of motion.

(b) If, $m = 4$ kg, $k_1 = 160$ N/m, $k_2 = 400$ N/m, $k_3 = 600$ N/m and $b = 48$ Ns/m, determine,

(i) the natural frequency and damping ratio of the system.

(ii) the free response of the system assuming that the initial conditions are : $x(0) = 0$ and $\dot{x}(0) = 32$ mm/s.

(iii) If the damper is now replaced by a critically damped damper, what would be the free response of the system? Assume that the initial conditions are: $x(0) = 8$ mm and $\dot{x}(0) = 20$mm/s.

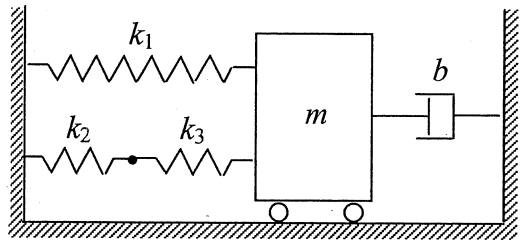


Fig. Q2

QUESTION 03

Fig. Q3 shows a two degree of freedom vibratory system consisting of a uniform circular disc of radius R and polar moment of inertia I, a mass m and three springs of stiffnesses k_1, k_2 and k_3.

(a) Determine the equations of motion for small oscillations and express them in the form of a matrix.

(b) Obtain the frequency equation (you do not need to simplify this equation).

(c) If, $k_1 = k_3 = k$, $k_2 = 2k$, and $I = 2mR^2$,

(i) show that the frequency equation can be expressed as

$$\omega^4 - \frac{9}{2}\left(\frac{k}{m}\right)\omega^2 + \frac{5}{2}\left(\frac{k}{m}\right)^2 = 0,$$

where ω is the natural frequency of vibration.

(ii) determine the natural frequencies of vibration of the system in terms of k and m.

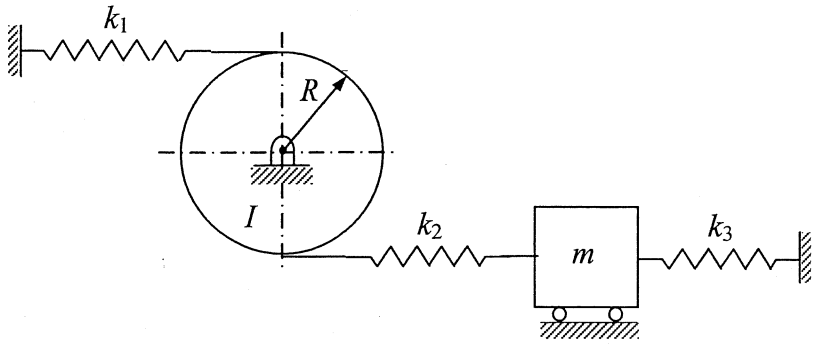


Fig. Q3

QUESTION 04

Fig. Q4 shows a spring-mass system consisting of two masses m_1 and m_2 and four springs of stiffnesses k, k, $2k$ and $3k$.

(a) Derive the equations of motion for small oscillations and obtain the frequency equation.

(b) If, $m_1 = m$ and $m_2 = 2m$,

(i) show that the frequency equation can be expressed as

$$\omega^4 - \frac{13}{2}\left(\frac{k}{m}\right)\omega^2 + 8\left(\frac{k}{m}\right)^2 = 0,$$

where ω is the natural frequency of vibration.

(ii) determine the natural frequencies of vibration of the system in terms of k and m and the ratio of amplitudes of motion of the two masses corresponding to each frequency.

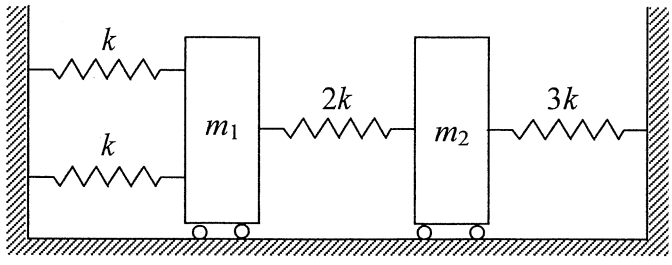


Fig. Q4

QUESTION 05

Fig. Q5 shows a torsional system consisting of three rotors, each of moment of inertia I and four shaft sections S_1, S_2, S_3 and S_4. The torsional stiffnesses of the shaft sections S_1, S_2, S_3 and S_4 are $2K$, K, K and $2K$ respectively.

(a) Derive the equations of motion for small torsional oscillations.

(b) Show that the frequency equation is given by

$$(\lambda-1)(\lambda-3)(\lambda-4)=0$$

where $\lambda=\dfrac{I\omega^2}{K}$ and ω is the natural frequency of torsional oscillations of the system.

(c) If, I = 2 kgm^2 and K = 4 Nm/rad, determine the natural frequencies of torsional oscillations of the system and the mode shapes corresponding to the lowest and the highest frequencies.

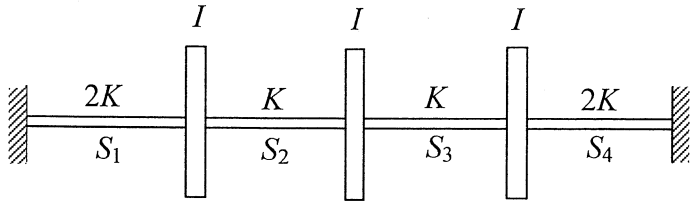


Fig. Q5

QUESTION 06

A mechanical system that is at rest is subjected to a unit step input $u(t)$. Its response is given by $y(t) = 2- 2e^{-2t}\cos 4t - e^{-2t}\sin 4t$. Determine the following.

(a) Transfer function of the system.

(b) Input-output differential equation of the system.

(c) Damped natural frequency, undamped natural frequency and the damping ratio.

(d) Response of the system to a unit impulse input.

(e) Steady state response for a unit step input.

QUESTION 07

(a) A mechanical system is given by the equations of motion,

$$\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}\ddot{y}_1\\ \ddot{y}_2\end{bmatrix}+\begin{bmatrix}2 & -1\\ -1 & 6\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}f_1(t)\\ f_2(t)\end{bmatrix}$$

Obtain the natural frequencies and the mode shapes of the system.

(b) The open loop transfer function of a unity, negative feedback control system is given by

$$G(s)=\frac{25}{s(s+5)}$$

Find the unit step response of the system using zero initial conditions.

QUESTION 08

(i) Explain why mechanical vibration is an important area of study for engineers.

Mechanical vibrations are known to have harmful effects as well as useful ones. Briefly describe three practical examples of good vibration and three examples of bad vibration.

(ii) Explain the advantages and disadvantages of the three basic maintenance strategies adopted in maintaining machinery and equipment.

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	**LAPLACE TRANSFORM** $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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