THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2013/2014

MPZ3231 – ENGINEERING MATHEMATICS IA

DURATION: THREE (03) HOURS

Date: 17th August 2014 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer **Five (05) question only**, selecting **at least two (02) questions from each section A & B.**
- This paper contains six (6) pages.
- Bold or underline letters represent the vectors.
- All symbols are in standard notation.

Section A

1. a) Given that $\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & \lambda & \mu \end{bmatrix}$

Find AB and hence determine λ and μ so that $\boldsymbol{B} = \boldsymbol{A}^{-1}$ [30%]

b) Let $\boldsymbol{A} = \begin{bmatrix} 1 & p & q \\ 0 & t & r \\ 0 & 0 & t \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & -pt & pr - qt \\ 0 & t & -r \\ 0 & 0 & t \end{bmatrix}$

where p, q, r and t are real numbers such that $t^2 = 1$.

Show that $\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}$,where $\boldsymbol{I}$ is the identity matrix of order 3. [30%]

Hence find a matrix $\boldsymbol{C}$ such that $\boldsymbol{C}\,\boldsymbol{A}^T = \boldsymbol{I}$ [20%]

c) Applying elementary raw/column operations, show that

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = (a-1)(b-1)(b-a)$$ [20%]

2. a) Consider the function $f(x) = \frac{x^2-7x+10}{x-1}$, x is real and $x \neq 1$

i. Find the horizontal, vertical and slant asymptotes of the curve of $y = f(x)$ [15%]

ii. Find $\frac{dy}{dx}$ and determine the sign of $\frac{dy}{dx}$ as x varies. [15%]

α) Find the coordinates of the turning points of the curve. [10%]

β) Sketch the graph $y = \frac{x^2-7x+10}{x-1}$. [10%]

γ) Deduce the graph of the function $g(x) = \frac{x^2-5x+4}{x}$, x is real and $x \neq 0$ [10%]

b) Let $u = r^5$ such that $r^2 = x^2 + y^2 + z^2$. Prove the following:

i. $\frac{\partial^2 u}{\partial x^2} = 5r^3\left\{r\frac{\partial^2 r}{\partial x^2} + 4\left(\frac{\partial r}{\partial x}\right)^2\right\}$, [10%]

ii. $r\frac{\partial r}{\partial x} - x = 0$, [05%]

iii. $r\frac{\partial^2 r}{\partial x^2} = 1 - \left(\frac{\partial r}{\partial x}\right)^2$, [05%]

Hence deduce that

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 30r^3$ [20%]

3. a) Let $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}^+$.and hence prove that result for $n \in \mathbb{Z}^-$. [10%]

b) i) Express $z^6 = -1$, in the form $z^6 = r\{\cos(2k\pi + \theta) + i\sin(2k\pi + \theta)\}$, where $r > 0$, $k \in \mathbb{Z}$ and $0 < \theta \leq 2\pi$. [05%]

ii) Let $w = \cos(\pi/6) + i\sin(\pi/6)$

Verify that w is a root of the equation $z^6 + 1 = 0$ and express the other roots in the form $\cos\theta + i\sin\theta$ and $x + iy$, where $x, y \in \mathbb{R}$. [35%]

iii) Deduce the four roots of the equation $z^4 - z^2 + 1 = 0$. [10%]

c) Find the loci of the following:

i. $\mathbb{R}e\left(\frac{z-2i}{z+2}\right) = 0$, [20%]

ii. $Im\left(\frac{z-2i}{z+2}\right) = 0$, [10%]

iii. $|z-2i| = |z+2|$. [10%]

4. a) Prove that the straight line $\frac{x-4}{3} = \frac{y-1}{2} = \frac{z-3}{1}$ intersects a line of intersection of the planes $x + y + 2z = 4$ and $3x - 2y - z = 3$.

and find the equation of the plane that contains these two lines. [40%]

b) Verify that the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{3}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ intersect each other. [30%]

c) Find the equation of the plane that passes through the points $(3,4,1), (1,1,-7)$ and $(2,2,-4)$. [30%]

Section B

5. a) Solve the differential equation:

$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$, where $y = 1$ when x = 1. [40%]

b) Solve the following differential equation by using an integrating factor

$\frac{dy}{dx} + y \tan x = \sin 2x$ [40%]

c) Solve the differential equation: $(3x^2 + y + 1)dx + (3y^2 + x + 1)dy = 0$ [30%]

6. a) Define the scalar product $\underline{a} \cdot \underline{b}$ and vector product $\underline{a} \times \underline{b}$ of the two vectors $\underline{a}$ and $\underline{b}$ [20%]

Given that A, B and C are three non collinear points and O is the origin, interpret the quantity $|\overrightarrow{OA} \times \overrightarrow{OB} . \overrightarrow{OC}|$, with the aid of a diagram. [30%]

b) The position vectors $\underline{a}, \underline{b}$ and $\underline{c}$ of the three points A, B and C with respect to the origin O are $\underline{i} + 2\underline{j} + 3\underline{k}$, $2\underline{i} + \underline{j} + \underline{k}$ and $\underline{i} - \underline{j} + 2\underline{k}$ respectively.

Find the following:

i) The cosine of the angle $\measuredangle ABC$. [10%]

ii) The area of the triangle ABC. [10%]

iii) The volume of the tetrahedron $OABC$. [10%]

iv) The value of $(\underline{b} \times \underline{c}) \times (\underline{c} \times \underline{a}) . (\underline{a} \times \underline{b})$. [20%]

7. a) The contents of three Urns are as follows.

Urn A : 6 red, 4 white balls

Urn B : 2 red, 6 white balls

Urn C : 1 red, 8 white balls

An Urn is chosen at random and two balls are drawn **without replacement** from this Urn. If both balls are red, find the probability that Urn A is being chosen. Find the probability that Urn C is being chosen. [50%]

b) Let x be a continuous random variable with probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(5-x) & if\ 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = \frac{6}{125}$ and find the mean and variance of the distribution.

8. a) By using the bisection method, show that a real root of the equation $f(x) = x^3 - x - 1 = 0$ is 1.3242 by considering 7 steps. [35%]

b) By using Newton- Raphson method, find the real root of the equation $f(x) = x^3 - x - 1 = 0$, correct to the three decimal places, with the initial guess $x_0 = 1.5$. [25%]

c) i) With usual notation and based on the information given, complete the following table.

x	$y = \tan x$	Δ	Δ^2	Δ^3	Δ^4
0.10	0.1003				
0.15	0.1511				
0.20	0.2027				
0.25	0.2553				
0.30	0.3093				

ii) By applying Newton's forward formula, find an approximate value for $\tan 0.12$ [40%]

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