**THE OPEN UNIVERSITY OF SRI LANKA**

**DIPLOMA IN TECHNOLOGY – LEVEL 03**

**FINAL EXAMINATION - 2013/2014**

**MPZ 3132 – ENGINEERING MATHEMATICS IB**

**DURARTION: THREE HOURS (3.00)**

**Date: 07th September 2014** **Time: 0930hrs – 1230hrs**

**Instructions:**

- **Answer only five (05) questions only, selecting at least one (01) question from Section B.**
- **Number of pages in the paper – 06.**
- **All symbols are in standard notation.**

**Important integrals**

$$\int e^{ax}\, sinbx dx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$$

$$\int e^{ax}\, cosbx dx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$$

$$\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Sign alternate $(+-+-+-+-+\ldots\ldots\ldots)$

$$\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \ldots\ldots\ldots\ldots$$

Sign alternate in pairs $(++--++--++--++--++--\ldots\ldots\ldots)$

$$\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \ldots\ldots\ldots\ldots$$

Sign alternate in pairs after the first term $(++--++--++--\ldots\ldots\ldots)$

# Section A

1)

I. State the Dirichlet's conditions and write down the Fourier series expansion of a period function $f(x)$. **[20%]**

II. Find the Fourier series representation of the function with period $2\pi$, defined by

$$f(t) = 1 + t, \qquad -\pi < t < \pi$$ **[30%]**

III. A periodic function of period 4 is defined as

$$f(x) = |x|, \quad -2 < x < 2.$$

Find its Fourier series expansion. Hence deduce that

$$f(x) = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left(\frac{(2n-1)x}{2}\right).$$ **[50%]**

2)

I. Using a suitable trial function, and find the general solution for the differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = x^2e^{3x}.$$ **[50%]**

II. Write down the Taylor series expansion of $f(x)$ about $x=0$.

If $y = \ln(\alpha + \beta Sin\theta)$, then prove that third Taylor polynomial about $\theta = 0$.

Hence, deduce the Taylor polynomial of $\ln\left[\frac{(\alpha+\beta\, Sin\theta)^2}{(\alpha-\beta\, Sin\theta)}\right]$ about $\theta = 0$.

**[50%]**

3)

I. Find the "Laplace transformation" of the following functions:

a) $\sinh(at)$,

b) $\cosh(at)$. **[20%]**

II. If $\mathcal{L}\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a); & t > a \\ 0 \quad ; & t \le a \end{cases}$, then show that

$$\mathcal{L}\{G(t)\} = e^{-as} f(s).$$ [45%]

Hence, find the Laplace transformation of the function:

$$e^{-3t}(4\cosh 6t - 7\sinh 6t).$$

III. Using the "Laplace transformation", solve the following differential equation.

$Y'' + Y = t$, $Y(0) = 1$, $Y'(0) = -2$. [35%]

4)

I. Let $Z$ be any complex number. Define arg $(Z)$. [05%]

a) Find the interaction points of the following curves.

$$Re(aZ) = |aZ - b| \text{ and } |Z| = \sqrt{2}\frac{b}{a}$$ [15%]

b) Shade the regions satisfying the following inequalities.

$$Re(aZ) \ge |aZ - b| \text{ and } |Z| \le \sqrt{2}\frac{b}{a}$$ [15%]

II. If $(Z) = \frac{1}{Z^2}$, $|Z| \le 2$, $0 \le \arg(Z) \le \frac{\pi}{2}$, $Z \in C$. Find the set $W = \{g(Z) / Z \in C\}$ and draw the $W$ – plane. [25%]

III. Prove that $\tan\left[i \ln\left(\frac{5-2i}{5+2i}\right)\right] = \frac{20}{21}$. [15%]

IV. Find the value of $(\sqrt{3} + i)^i$. [15%]

5)

I.

a) Define the Poisson distribution and estimate its expectation and variance. [30%]

b) Births in a hospital occurs randomly at an average rate of 18 births per hour. What is the probability at observing more than or equal to 2 births in a given hour of the hospital?. [15%]

II. The time taken to assemble a car in a certain plant is a random variable having normal distribution with a mean of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assemble at this plant in a period of time:

a) lest than 19.5 hours? **[15%]**

b) between 20 and 22 hours? **[15%]**

c) Find the value of hours (h) such that $P(|X-20|<h)=0.75$. Hence, find the limit, with which the central $75\%$ of the distribution. **[25%]**

6)

I. Differentiate with respect to $t$,

$$\left[\frac{(\underline{r}\times\underline{a})}{r}-\frac{\underline{r}}{(\underline{a}\cdot\underline{r})}\right],$$

where $\underline{r}$ is vector function of $t$, $r$ is length of $\underline{r}$ and $\underline{a}$ is constant vector. **[20%]**

II. A particle moves in a plane with a constant acceleration of $10\,ms^{-2}$ in the direction $OX$. Initially the particle is at the point whose position vector is $2\underline{i}+3\underline{j}$ with a velocity of $12ms^{-1}$ in the direction $OY$. Find the path of the particle at any time $t$. **[25%]**

III. Let $\underline{r}=\underline{p}\sin\alpha t+\underline{q}\cos\alpha t-\frac{7\underline{s}}{\alpha^2}\sin\alpha t$. Prove that

$\frac{d^2\underline{r}}{dt^2}+\alpha^2\underline{r}+\frac{14\underline{s}}{\alpha}\cos\alpha t=0$, where $\underline{p}$, $\underline{q}$, $\underline{s}$ are constant vectors and $\alpha$ is a constant scalar. **[20%]**

IV. A particle P moves on a plane. At time t, the polar coordinates of P is $(r,\theta)$ and the position vector OP is $\underline{r}$. If the unit vector along $r$ is $\underline{l}$ and the unit vector perpendicular to $\underline{r}$ is $\theta$ and increasing direction is $\underline{m}$.

Prove that the velocity of the particle at time t is $\underline{v}=\dot{r}\underline{l}+r\dot{\theta}\underline{m}$ and acceleration is

$$\underline{f}=(\ddot{r}-r\ddot{\theta})\underline{l}+\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\underline{m}.$$ **[35%]**

# Section B

7)

I. Find the moment of inertia of the following bodies about the given axis.

a) A uniform circular ring mass ***m*** and radius ***r***, the axis perpendicular to the plane of the ring passing through the center. **[20%]**

b) A uniform circular disc mass ***m*** and radius ***r***, the axis perpendicular to the plane of the disc passing through the center. Find the moment of inertia of the disc about its diameter. **[25%]**

II. A uniform circular plate of radius ***a*** has a mass M. A hole of radius $^{a}/_{5}$ is punched in the plate; the center of the hole being a distance of $^{3a}/_{5}$ from the center of the plate. Find the moment of inertia of the plate about,

a) a diameter through the center of the hole **[30%]**

b) a line through the center of the disc perpendicular to its plane. **[25%]**

8).

I. The water face of a dam is in the form of trapezium. The bottom width is $l$ meter and the top width is $(l + t)$ meters. The face dam is vertical and the depth of water is $H$ meters. When the water level is par with the top level of the dam,

show that

a) The resultant thrust on the dam is $\frac{\rho H^2}{6}(3l + t)$ N, **[40%]**

b) The depth of the centre of pressure is $\frac{1}{2}\left(\frac{4l + t}{3l + t}\right)H$ meters. **[20%]**

where the density of the water is $\rho\ kgm^{-3}$.

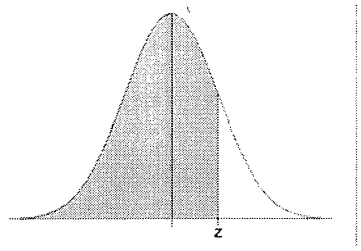
II. The cross-section of a horizontal canal is trapezium width *1 m* and *2 m* at the bottom and at the top respectively. The depth of the canal is *1 m*. The canal is closed by a vertical door which is pivoted at the upper edge and fasten at the middle point of the lower edge by application of a horizontal force $F$ perpendicular to the door. Show that, when the canal is full of water, $F = \frac{5\rho}{12} N$. where $\rho$ being the density of water.

**[40%]**

END

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# Standard Normal Cumulative Probability Table




**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |