THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY – LEVEL 03
FINAL EXAMINATION - 2014/2015
MPZ 3132 – ENGINEERING MATHEMATICS IB
DURARTION: THREE HOURS (3.00)

Date: 08th August 2015 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer five (05) questions only.**
- **Number of pages in the paper – 06.**
- **All symbols are in standard notation.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ ..... [Sign alternate $(+ - + - + \cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $--++--++--\cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(++--++--++--\cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

1)

I. State the Dirichlet's conditions and write down the Fourier series expansion function $f(x)$ with period $-\pi$ to π. **[20%]**

II. Find the period of each the following functions.

a) $\sin 3x$,

b) $\cos 5x$ **[20%]**

III. Find the Fourier series representation of the function with period 2π, defined by

$$f(x) = \begin{cases} x(x+1), & x \in (-\pi, \pi) \\ f(x+2k\pi), & otherwise, k \epsilon \mathbb{Z} \end{cases}$$

Using the parseval's formula deduces that $\sum_{n=1}^{\infty}\left(\frac{4+n^2}{n^4}\right) = \frac{\pi^2}{90}(15 - 11\pi^2)$.

[60%]

2)

a) Define the n^{th} order Taylor polynomial of $f(x)$ about $x = a$.

b) Let $f(x) = \frac{1-x^2}{(1-2xcos\theta+x^2)}$, where θ is a constant. Show that

$(1 + x^2 - 2xcos\theta)\frac{df^5(x)}{dx^5} + 10(x - cos\theta)\frac{df^4(x)}{dx^4} + 20\frac{df^3(x)}{dx^3} = 0,$

and deduce that the Taylor polynomial of $f(x)$ about $x = 0$ is

$1 + \sum_{r=1}^{5} 2\cos(r\theta)x^r$.

[60%]

c) Prove that Taylor series expansion of $(1/x)$ about $x = 3$ is $\sum_{r=0}^{\infty}\left(\frac{(-1)^r(x-3)^r}{3^{r+1}}\right)$,

and hence show that the series convergence to $(1/x)$ and also find the radius of convergence. **[40%]**

(Hint: use the convergence of a geometric series)

3)

I. Using a suitable trial function:

a) Find a particular integral for the following differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = x^2e^{5x}.$$

b) Find the general solution of the above differential equation.

[50%]

II. Solve the following homogeneous differential equations:

$$\frac{dx_1}{dt} = x_1 + 2x_2 - x_3,$$

$$\frac{dx_2}{dt} = x_1 + x_3,$$

$$\frac{dx_3}{dt} = 4x_1 - 4x_2 + 5x_3\text{, where } x_1, x_2 \text{ and } x_3 \text{ are function of } t. \quad \textbf{[50\%]}$$

4)

I. Define the "Laplace transformation" of $F(t)$. **[05%]**

II. Find the "Laplace transformation" of each the following functions: **[30%]**

a) $\sin(at)$

b) $\sin^3(2t)$.

III. Using that shift theorem, deduce that $\mathcal{L}\{e^{3t}\sin 5t\} = \frac{5}{(s-3)^2+25}$. **[15%]**

IV. Find the inverse "Laplace transformation" of the following: **[20%]**

a) $\frac{2s^4-5s^3+4s^2-1}{s^5}$

b) $\frac{5s+3}{(s-1)(s^2-2s+5)}$.

V. Using the "Laplace transformation", solve the following differential equation:

$y'' + 9y = \cos 2t, \quad y(0) = a, \quad y'(0) = b.$ **[30%]**

5)

I.

a) Define the Binomial distribution and estimate its expectation and variance. [30%]

b) Ten present of computer parts produced by a certain supplier are defective. What is the probability that a sample of 10 parts contains more than 3 defective ones? [15%]

II. It is known that in a sack of mixed paddy seeds 35% are red. Use the normal approximation to the binomial distribution to find the probability that in a sample of 400 seeds there are

a) lest than 120 red seeds. [20%]

b) between 120 and 150 red seeds. [20%]

III. For a certain type of computer, the length of time between charges of the battery is normal distributed with a mean of 50 hours and a standard deviation of 15 hours. What is the probability that the length of time will be between 50 and 70 hours. [15%]

6)

I.

a) Shade the complex region satisfying each of the following inequality:

$|Z-2+i| \leq 1$ and $|2Z+3| \geq 4$. [20%]

b) Show that the function $f(Z) = \frac{3Z+4}{Z-1}$, $Z \neq 1$ is one to one and find its inverse function. [20%]

c) Express $\log(\sqrt{3}-i)$ in the form of $x+iy$ where x and y are real numbers. [15%]

II.

a) A particle moves along the curve $x = 5t^2 - 2t$, $y = t^3 - 4t^2$ and $z = 3t - 6$, where t is time. Find the components of its velocity and acceleration at time $t = 2$ in the direction $2\underline{i} - 3\underline{j} + 6\underline{k}$. [30%]

b) A force of magnitude $40N$, acting in the direction of $6\underline{i} - 8\underline{j}$, moves a particle from $P(-2, 4)$ to $Q(7, 10)$. Calculate the work done by the force.

[15%]

7)

I. Find the moment of inertia of the following bodies about the given axis.

a) A uniform rod AB of mass m and length $2l$, the axis perpendicular to the rod passing through the midpoint of the rod. **[20%]**

b) A uniform circular disc of mass M and radius l, the axis perpendicular to the plane of the disc passing through the center. **[20%]**

II. A pendulum consists of a uniform rod AB, of mass m and length $2l$ fixed at B to a point on the circumference of a uniform circular disc of mass M and radius l. The disc and the line of the rod lie in the same vertical plane with the center of the disc lying on the rod produced. The pendulum is free to rotate in a vertical plane about a fixed horizontal axis through A perpendicular to the plane of the disc.

a) Find the moments of inertia of the pendulum about the axis perpendicular to the plane of the disc passing through A. **[30%]**

b) When the rod is horizontal the disc is projected with an angular velocity ω in the downward vertical direction. Find the angular velocity after the rod turns an angle θ. If the system rotates complete circles prove that

$$\omega^2 \geq \frac{12g(m+3M)}{l(8m+57M)}$$ **[30%]**

8).

I. A circular plate of radius a is immersed in a homogeneous liquid of density ρ so that its plane is vertical and highest point A is on the free surface of the liquid. Show by integration, that

a) the liquid thrust on the plate is $\pi a^3 \rho g$,

b) the center of pressure is at depth $\frac{5a}{4}$ vertically below A.

[60%]

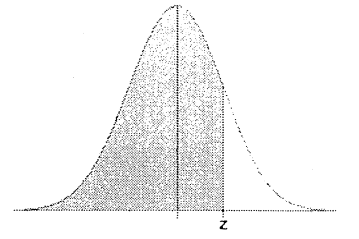
II A uniform rectangular lamina *ABCD* is immersed vertically in a homogeneous liquid such that *AB* and *CD* are horizontal. The distance to *AB* and *CD* from the surface of the liquid are l_1 and l_2 respectively. Prove that the center of pressure of the lamina is at the distance

$\frac{2}{3}\left(\frac{l_1^2 + l_1 l_2 + l_2^2}{l_1 + l_2}\right)$ from the free liquid surface. **[40%]**

END

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Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998