THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2014/2015

MPZ3231 – ENGINEERING MATHEMATICS IA

DURATION: THREE (03) HOURS

Date: 30th August 2015 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer **Five (05) question only**, selecting **at least two (02) questions from each section A & B**.
- This paper contains four (4) pages.
- Bold letters represent the vectors.
- All symbols are in standard notation.

Section A

1. a) Let $f(x) = x^2 - 2x + 2$ and $\boldsymbol{A} = \begin{pmatrix} -1 & -5 \\ 1 & 3 \end{pmatrix}$.

   i) Show that $f(\boldsymbol{A}) = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero matrix of order 2.

   ii) Find $f(1-i)$,where $i^2 = 1$ and deduce that $(1-i)^6 = 8i$.

   ii) Find the real constants α and β such that $x^6 = f(x)g(x) + \alpha x + \beta$ where $g(x)$ is a suitable polynomial.

   iv) Deduce that $\boldsymbol{A}^6 = 8\begin{pmatrix} 2 & 5 \\ -1 & -2 \end{pmatrix}$. [40%]

   b) Let $\boldsymbol{B} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & -1 \\ 1 & 2 & -2 \end{bmatrix}$.

   i) Find $\boldsymbol{B}^2$ and $\boldsymbol{B}^3$ and hence find α, β and γ such that $\boldsymbol{B}^3 = \alpha\boldsymbol{B}^2 + \beta\boldsymbol{B} + \gamma\boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix of order 3.

   ii) Express $\boldsymbol{B}^{-1}$ in terms of $\boldsymbol{B}^2, \boldsymbol{B}$ and $\boldsymbol{I}$ and write in the matrix form. [40%]

   c) Show that $\begin{vmatrix} 1 & y & 2y-1 \\ y+1 & 2y & 2 \\ y+2 & 3y & y^2+2y \end{vmatrix} = -y(y+1)(y-1)^2$ [20%]

2. a) Consider the function $y = f(x) = \frac{(3x-4)(x+1)}{(x-2)}$.

i) Find the horizontal, vertical and slant asymptotes of the curve $y = f(x)$. [15%]

ii) Find $\frac{dy}{dx}$ and determine the sign of $\frac{dy}{dx}$ as x varies. [15%]

α) Find the coordinates of the turning points of the curve. [15%]

β) Sketch the graph of the above function $f(x)$. [15%]

γ) Deduce the graph of the function $y = g(x) = \frac{(3x+2)(x+3)}{x}$ [10%]

b) Let $z = (x^2 + y^2)\tan^{-1}\frac{x}{y}$. Show that

i) $\frac{\partial z}{\partial y} = 2y\tan^{-1}\frac{x}{y} - x$. [10%]

ii) $\frac{\partial z}{\partial x} = 2x\tan^{-1}\frac{x}{y} + y$. [05%]

iii) $\frac{\partial^2 z}{\partial y^2} = 2\tan^{-1}\frac{x}{y} - \frac{2xy}{x^2+y^2}$. [05%]

iv) $\frac{\partial^2 z}{\partial x^2} = 2\tan^{-1}\frac{x}{y} + \frac{2xy}{x^2+y^2}$. [05%]

v) $\frac{\partial^2 z}{\partial x \partial y} = \frac{y^2 - x^2}{x^2 + y^2}$. [05%]

3. a) i) Using De Moivre's theorem, show that

$(1 + i\tan\theta)^n + (1 - i\tan\theta)^n = \frac{2\cos n\theta}{\cos^2\theta}$ where $n \in \mathbb{Z}^+$ and $\cos\theta \neq 0$. [10%]

ii) Using result (i), show that the roots of the equation $(1+z)^4 + (1-z)^4 = 0$ are

$\pm i\tan\frac{\pi}{8}, \pm i\tan\frac{3\pi}{8}$ and

Hence show that $\tan^2\frac{\pi}{8} = 3 - 2\sqrt{2}$. [40%]

b) Interpret geometrically each of the following equations.

i) $|z + 4i|^2 - |z - 4i|^2 = 10$. [20%]

ii) $|z + 2| = |z - 2i|$. [15%]

iii) $\text{Arg}(z - 2i) = \frac{\pi}{6}$. [15%]

4. a) $ABCD$ is a rectangular tetrahedron and X is the mid - point of CD. Prove that the angle between AX and BC is $\cos\left(\frac{\sqrt{3}}{6}\right)$. [30%]

   b) Find the intersecting line of the planes $15x - 34y + 21z - 10 = 0$ and $9x - 20y + 12z - 5 = 0$ in the symmetrical form. [30%]

   c) i) Find the equation of the normal to the hyperbola $xy = c^2$ at the point $A = \left(ct, {}^{c}/_{t}\right)$.
   ii) If this normal meets the curve at the point B, find the coordinates of B.
   iii) Show that the locus of the mid - point of AB is $c^2(x^2 - y^2) + 4x^3y^3 = 0$. [40%]

Section B

5. a) Solve the differential equation:
   $x^2\frac{dy}{dx} = y^2 + 5xy + 4x^2$, given that $y = 1$ when $x = 1$. [40%]

   b) Solve the following differential equation by using an integrating factor:
   $3e^{3x}y - 2xe^x + e^{3x}\frac{dy}{dx} = 0$. [30%]

   c) Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$. [30%]

6. a) The position vectors of A, B and C with respect to the origin O are respectively
   $-2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $-4\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $-3\boldsymbol{i} - 3\boldsymbol{k}$
   i) Prove that OA is perpendicular to AB and AC.
   ii) Find the angle between OB and OC.
   iii) Find the area of the triangle ABC. [40%]

   b) Show that the four points, whose position vectors are $2\boldsymbol{i} - 4\boldsymbol{j} - 3\boldsymbol{k}$, $-3\boldsymbol{i} - \boldsymbol{j} - 6\boldsymbol{k}$, $-7\boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}$ and $-2\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k}$ are coplanar. [30%]

   c) Let $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ be any three vectors and let $[\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}]$ denotes the scalar triple product $\boldsymbol{a} \times \boldsymbol{b}.\boldsymbol{c}$, Prove that $[\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{b} + \boldsymbol{c}, \boldsymbol{c} + \boldsymbol{a}] = 2[\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}]$ [30%]

7. a) During an epidemic of a certain disease a doctor is consulted by 330 people suffering from symptoms commonly associated with that disease. Of the 330 people 135 are female, of whom 60 actually have the disease and 75 do not. 45 males have the disease and the rest do not.

   i) A person is selected at random. The event that this person is a female is denoted by F and the event that this person is suffering from the disease is denoted by D.

   Evaluate:

   $\alpha)\ P(F)$ $\beta)\ P(F \cup D)$ $\gamma)\ P(F \cap D)$ $\delta)\ P(F/D)$ [40%]

   ii) If three different people are selected at random without replacement, what is the probability that

   $\alpha)$ All three having the disease

   $\beta)$ Exactly one of the three having the disease. [30%]

   b) A bunch of seven keys contains exactly one key which will open a certain lock. Keys are selected at random, without replacement, until the correct key is found.

   Let X denotes the number of keys which are tried, including the correct one. Find the probability distribution of X and show that its mean is 4 and find its variance. [30%]

8. a) Find a root of the equation $x^3 - 3x - 6 = 0$ by using the Newton - Raphson method correct to three decimal places [40%]

   b) Compute the value $\int_0^1 \frac{dx}{1+x^2}$ with 8 strips by using the

   i) Trapezoidal rule,

   ii) Simpson rule, and hence find an approximate value for π. [60%]

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