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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2016/2017)
MPZ3231: Engineering Mathematics IA

Date: 21st November 2017 (Tuesday) **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- *This paper consists of* ***Section – A*** *and* ***Section – B.***
- *Selecting at* ***least two questions*** *in each section, answer* ***five (05)*** *questions only.*
- *Number of pages in the paper is eight (05).*
- *All the symbols are in standard notation unless they are defined.*

Section-A

Q1. (a). (i) State the Ceyley-Hamilton's theorem. (05%)

(ii). Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -3 \\ 0 & 1 & 3 \end{pmatrix}$.

(α) Find A^2. (10%)

(β) Using the characteristic equation of A, prove that (20%)
$A^3 - 3A^2 + 3A - \bar{I} = \bar{0}$, where $\bar{I}$ and $\bar{0}$ are the unit matrix and the zero matrix of order three. Hence, find a non-zero matrix B such that $B^3 = \bar{0}$.

(γ) Find only the constants k, l and m such that (35%)
$x^n = (x-1)^3 f(x) + kx^2 + lx + m$, where n is a positive integer such that $n > 2$ and $f(x)$ is a polynomial of degree $n-3$.
Hence, deduce that for every $n \in \mathbb{Z}^+$

$$A^n = \begin{pmatrix} \frac{1}{2}(n-1)(n-2) & \frac{1}{2}n(n-1) & \frac{1}{2}n(n+1) \\ n(2-n) & 1-n^2 & -n(n+2) \\ \frac{1}{2}n(n-1) & \frac{1}{2}n(n+1) & \frac{1}{2}(n+1)(n+2) \end{pmatrix}.$$

(b). Using the column operations and row operations, prove that (30%)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\frac{(a+b+c)}{2}[(a-b)^2+(b-c)^2+(c-a)^2].$$

Q2. (a). Let $f(x) = \frac{x^3}{1-x^2}$, where $x \neq \pm 1$.

(i). Find the vertical and slant asymptotes of the curve of $y = f(x)$. (20%)

(ii). Prove that $\frac{dy}{dx} = -\frac{x^2(x-\sqrt{3})(x+\sqrt{3})}{(1-x)^2(1+x)^2}$. (05%)

(iii). Determine the sign of $\frac{dy}{dx}$ as x varies from $-\infty$ to ∞. (10%)

(iv). Find the turning points of the curve of $y = f(x)$. (10%)

(v). Sketch the graph of $y = f(x)$ and hence deduce the graph of $y = |f(x)|$. (25%)

(b). If $z(x+y) = x^2 + y^2$, then prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. (30%)

Q3. (a). (i). Using the De Moivre's theorem, find $sin5\theta$ in terms of $sin\theta$ and $cos5\theta$ in terms of $cos\theta$. Hence prove that (35%)

$$\frac{tan5\theta}{tan\theta} = \frac{tan^4\theta - 10tan^2\theta + 5}{5tan^4\theta - 10tan^2\theta + 1}, \text{ where } \theta \neq n\pi \text{ and } n \in \mathbb{Z}.$$

(ii). Considering the general solutions of the equation $tan5\theta = 0$, determine all the roots of the equation $x^4 - 10x^2 + 5 = 0$. Hence, find the roots of the equation $x^2 - 10x + 5 = 0$. Deduce that $sec^4\frac{\pi}{5} + sec^4\frac{2\pi}{5} = 112$. (30%)

(b). Let θ be constant such that $0 < \theta < \frac{\pi}{2}$ and $r \in \mathbb{R}^+$. P denotes the point corresponding to $z \in \mathbb{C}$ that sitisfies the locus $arg\left(\frac{z-r}{z+r}\right) = \theta$.

(i). Using the Argand diagram, show that P does not lie on real axis and lower half plain. (15%)

(ii). Sketch the locus of P. (10%)

(iii). Deduce that the above curve can be expressed as $|z - ircot\theta| = rcosec\theta$ with $Im(z) > 0$. (10%)

Q4. (a). Prove that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P\ (at^2, 2at)$ is $y + tx = 2at + at^3$. (15%)

The normal to the parabola $y^2 = 4ax$ at $P\ (at^2, 2at)$ meets the x −axis of at G. The point Q lies on the extended line GP such that $GP = PQ$.

(i). Find the coordinates of the point G. (05%)

(ii). Find the coordinates of the point Q. (10%)

(iii). Show that the locus of Q is the parabola $y^2 = 16a\ (x + 2a)$. (15%)

(b). Find the equations of two straight lines through the origin each of which intersects the straight line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ and inclined at an angle of $\frac{\pi}{3}$ to it. (55%)

Section-B

Q5. (a). Using an integrating factor, find the general solution of the differential equation $\frac{dy}{dx} + ycotx = e^x$. (30%)

(b). Using the substitution $y = vx$, find the general solution of the differential equation $2xy\frac{dy}{dx} = x^2 + y^2$. (30%)

(c). Prove that the general solution of the differential equation (40%)

$\frac{d^2y}{dt^2} + 2a\frac{dy}{dt} + (a^2 + \omega^2)y = 0$ is $y = e^{-at}(Acos\omega t + Bsin\omega t)$,

where A and B are arbitrary contants. If $y(0) = 4$ and $y^1(0) = -4a$, then find the solution.

Q6. (a). Prove that the equation of a straight line passing through a given point with the position vector $\boldsymbol{a}$ and parallel to a given vector $\boldsymbol{b}$ is $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where λ is a parameter. (25%)

Let L be the line corresponding to $\boldsymbol{a} = \boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$.

Prove that the equation of L is $\frac{x-1}{1} = \frac{y-6}{-2} = \frac{z+3}{2}$.

(b). Prove that the vector equation of a plane passing through a given point with position vector $\boldsymbol{a}$ and perpendicular to the vector $\boldsymbol{n}$ is $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$. (25%)

Let P be the plane corresponding to, $\boldsymbol{a} = 3\boldsymbol{i}$ and $\boldsymbol{n} = 2\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}$.

Prove that equation of P is $2x - 3y + 6z - 6 = 0$.

(c). If the line L and the plane P intersect at the point A,

(i). Find the position vector of A. (15%)

(ii). Find the acute angle between L and P. (15%)

(iii). Find the vector equation of the line that lies on P passes through A and perpendicular to L. (20%)

Q7. (a). Let $f(x) = x^3 - x - 3$, where $x \in \mathbb{R}$. Prove that $f(x) = 0$ has a real root α that lies between 1 and 2. (15%)

(b). Let $g(x_k) = x_k^3 - 3$.

(i). Show that if $x_{k+1} = g(x_k)$, then it is an iterative scheme for iterating an approximation value of the root α. (10%)

(ii). Taking $x_0 = 1 \cdot 5$, find four iterations to correct to four decimal places. (10%)

(iii). Discuss the convergence of the iteration in the interval $[1, 2]$ using $g'(x)$. (15%)

(c). Let $h(x_k) = \sqrt[3]{(x_k + 3)}$.

(i). Show that if $x_{k+1} = h(x_k)$, then it is an iterative scheme for iterating an approximation value of the root α. (10%)

(ii). Taking $x_0 = 1 \cdot 5$, find twelve iterations to correct to nine decimal places. (20%)

(iii). Discuss the convergence of the above iteration in the interval $[1, 2]$ using $h'(x)$. (15%)

(d). Deduce an approximation value for the root α to correct to nine decimal places. (05%)

Q8. (a). (i). State Bayer's theorem. (10%)

(ii) A bag contains n number of balls that are identical, but different in colour. Also, nobody knows how many white balls are in the bag. If a one ball was randomly drawn from the bag, then it was white. Find the probability that the bag contains only white balls. (40%)

You may assume that $\sum_{i=0}^{n} i = \frac{1}{2}n(n+1)$.

(b). The amount of rice (in hundred kilograms) that a supermarket sells in a day is a random variable with density function $f(x) =$ (50%)

$\begin{cases} k(3-|x-3|), & 0 \le x \le 6 \\ 0, & \text{Otherwise} \end{cases}$, where k is a constant.

(i). Find the value of k such that $f(x)$ is a probability density function.

(ii). What is the probability that the number of kilograms of rice that will be sold in a day is,

(α). More than 300 kilograms?

(β). Between 150 and 450 kilograms?

(iii). Denote by A and B the events in (α) and (β), respectively. Determine whether the events A and B are independent.

End

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