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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2016/2017)
MPZ4230: Engineering Mathematics II

Date: 22nd November 2017 (Wednesday) **Time: 9:30 am – 12:30 pm**

Instructions:

- Answer only six (6) questions.
- State any assumptions you required.
- This paper contains five (5) pages.
- Show all your workings.
- All symbols are in standard notation.

Q1.

a) In surveying a triangular plot of land two of its sides were measured and found to be $a = 160\ m$ and $b = 210\ m$ in an error of at most $0.1\ m$ in either measurement and the angle between a and b was $C = \frac{\pi}{3}$ exactly. The length of the third side c is $(a^2 + b^2 - 2ab\cos C)^{\frac{1}{2}}$

i. Using appropriate Taylor polynomial estimate the value of the third side. [15%]

ii. Estimate the maximum error in calculating the length of the third side. [15%]

b) Find stationary points of $f(x,y) = x^4 + y^4 - 36xy$. [30%]

c) Let $\underline{F}(x,y) = x^2\underline{i} + y\,\underline{j}$

i. Is this vector field conservative? [10%]

ii. Evaluate $\int_{(2,0)}^{(0,2)} \underline{F} \cdot dr$ along the straight line from (2,0) to (0,2). [20%]

iii. Evaluate $\int_{(2,0)}^{(0,2)} \underline{F} \cdot dr$ along the quarter circle center at the origin and radius 2 between (2,0) to (0,2). [10%]

Q2.

a) i. State the Necessary condition for the function $f(z) = u + iv$ to be analytic.[10%]

ii. Show that the function $u(x, y) = \cos 2x \cosh 2y$ is a harmonic function. [15%]

iii. Derive the analytic function of the form $f(z) = u + iv$, where v is complex conjugate of $u(x, y) = \cos 2x \cosh 2y$. [30%]

iv. Express $f(z)$ of part (c) in terms of z. [15%]

b) Evaluate $\oint_C \frac{z}{z^2+9} dz$ where c is the path $|z + 3i| = 3$. [30%]

Q3.

The number of grams of a given salt which dissolved in 100g of water at different temperatures (^{0}C) is shown below.

Temperature (^{0}C)	10	20	30	40	50	60	70	80
Weight (g)	53.5	59.5	65.2	70.6	75.5	80.2	85.5	90.0

a) Identify the independent variable and dependent variable. [05%]

b) Constrict the scatter plot for this data. [10%]

c) Find the best fit line for the data given. [25%]

d) Interpret the slope coefficient of the regression line. [05%]

e) Predict the average weight of salt which would dissolve at temperature 25^{0}C. Comment on your result. [10%]

f) Following table represents some summary statistics for simple linear regression model for the above data. Fill in the blanks of the following table. [25%]

Model	Sum of Squares (SS)	Degree of Freedom (df)	Mean Square (MS)	F- statistic
Regression	1128.412	1	.....	.....
Residual	.....	.....	0.338	
Total	.....	7		

g) Test the significance of the regression model at 5% level of significance using above table. (Hint: Use F test) [20%]

Q4.

a) A company manufactures bars of soap. In a random sample of 70 bars, 18 were found to be miss-shaped.

i. Calculate an approximate 95% confidence interval for the proportion of miss-shaped bars of soap. [25%]

ii. Interpret this confidence interval. [05%]

b) A software company markets a new computer game with two experimental packaging designs. Design 1 is sent to 11 stores; their average sales for the first month is 52 units with sample standard deviation 12 units. Design 2 is sent to 6 stores; their average sales for the first month is 46 units with sample standard deviation 10 units. Decision maker wants to know that the sales per month of the two designs are different or not.

i. State the hypothesis that you would test. [10%]

ii. Test the above hypothesis using a suitable method. (Use 5% level of significance) (Hint: we don't have evidence to assume that two variances are equal) [55%]

iii. State the assumptions that you have made. [05%]

Q5.

a) Consider the differential equation $\frac{dy}{dx} = (1 + x^2)y$

i. Using fourth order Taylor's series method, find $y(0.1)$ and $y(0.2)$ considering step size 0.1 with initial condition $y(0) = 1$. [45%]

ii. Find the actual value of $y(0.1)$and $y(0.2)$. [15%]

b) i. State the fourth order Runge – Kutta formula for solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$. [10%]

ii. Let $\frac{dy}{dx} = 1 + y^2 \; ; y(0) = 0$. Find $y(0.2)$ and $y(0.4)$ by using Runge-Kutta fourth order method. [30%]

Q6.

a) Classify the elliptic, parabolic and hyperbolic form of the second order partial differential equation. [10%]

b) Show that the explicit formula for numerical solution of hyperbolic partial differential $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ is $u_{i,j+1} = a^2 r^2 u_{i+1,j} + 2(1 - a^2 r^2) u_{i,j} + a^2 r^2 u_{i-1,j} - u_{i,j-1}$, with $r = \frac{k}{h}$, where h is step size in the $x-$ direction and k is step size in the $t-$ direction. [30%]

c) Using the above formula solve

$$\frac{\partial^2 u}{\partial x^2} = 4 \frac{\partial^2 u}{\partial t^2} \qquad 0 \le x \le 1, \quad 0 \le t$$

With boundary conditions

$$u(0,t) = u(1,t) = 0, \qquad 0 \le t$$

and the initial conditions

$$u(x,0) = 0 \ , \qquad 0 \le x \le 1$$

$$\frac{\partial u}{\partial t}(x,0) = \sin 4\pi x \qquad 0 \le x \le 1$$

Find solution for five steps of t using the forward difference method with $h = 0.1$. [60%]

Q7.

Consider the series RLC circuit, inductance $(L) = 1$ Henry, resistance $(R) = 40$ ohms, capacitance $(C) = 16\times10^{-4}$ farad and voltage $\big(E(t)\big) = 100\cos 10t$ volts. Consider initial current and charge are zero.

a) Find the charge and current at time t. [90%]

b) Indicate steady state solution of charge. [10%]

Assume that,

Kirchhoff's voltage law for series circuit yield

$$V_L + V_R + V_C = E(t), \quad \text{where}$$

$$V_L = L\frac{di}{dt}, \quad V_R = iR, \quad V_C = \frac{1}{C}\int i\,dt, \quad i = \frac{dq}{dt}$$

Q8.

a) Show that the mapping $L(x, y, z) = (x + y + z,\ x + 2y + z,\ x + 3y + 2z)$ is a linear transformation. [20%]

b) Consider the basis $S = \left(\underline{v_1}, \underline{v_2}, \underline{v_3}\right)$ for $V_3(\mathbb{R})$, where $\underline{v_1} = (1,1,1), \underline{v_2} = (1,2,1)$ and $\underline{v_3} = (1,3,2)$. Let $T : V_3(\mathbb{R}) \to V_3(\mathbb{R})$ be a linear transformation for which $T\left(\underline{v_1}\right) = (3,4,5),\ T\left(\underline{v_2}\right) = (4,6,8)$ and $T\left(\underline{v_3}\right) = (6,9,12)$.

i. Find the formula for $T(x_1, x_2, x_3)$. [35%]

ii. Find $T(1,2,3)$. [10%]

c) Consider the following linear transformation $L_1 : M_{2\times2} \to M_{2\times2}$,

$$L_1\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} p+q+r & 0 \\ p+2q+r & p+3q+2r \end{bmatrix}$$

i. Find a basis for $\ker L_1$ and a basis for range L_1. [25%]

ii. Verify that the theorem $\dim[\ker L_1] + \dim[\text{range } L_1] = \dim L_1$. [10%]

Q9.

a) Let $q(x_1, x_2, x_3) = {x_1}^2 + {x_2}^2 + {x_3}^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$ be a quadratic function of x_1, x_2 and x_3. Determine the symmetric matrix A of the above quadratic form. [10%]

b) Find the eigen values of A and its corresponding eigen vectors. [30%]

c) Find an orthogonal matrix P such that $D = P^{-1}AP$, where D is a diagonal matrix. [25%]

d) Using above results find D^n, where n is a positive integer. Is D^{-1} exist? Justify your answer. [15%]

e) Derive the quadratic functions of $Q\left(\underline{x}\right) = \underline{x}^T A \underline{x}$ and $Q\left(\underline{y}\right) = \underline{y}^T D \underline{y}$

where $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ [10%]

f) Show that $P^{-1} = P^T$, and hence show that $Q\left(\underline{x}\right) = Q\left(\underline{y}\right)$, where $\underline{y} = P^T\underline{x}$. [10%]

-END-

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Probabilities for the standard normal distribution

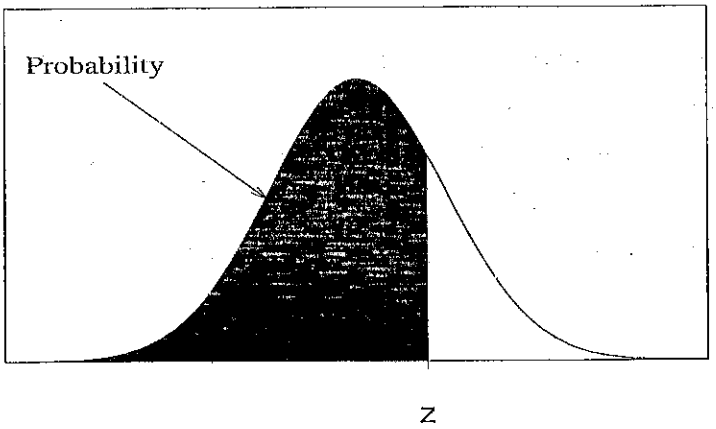


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

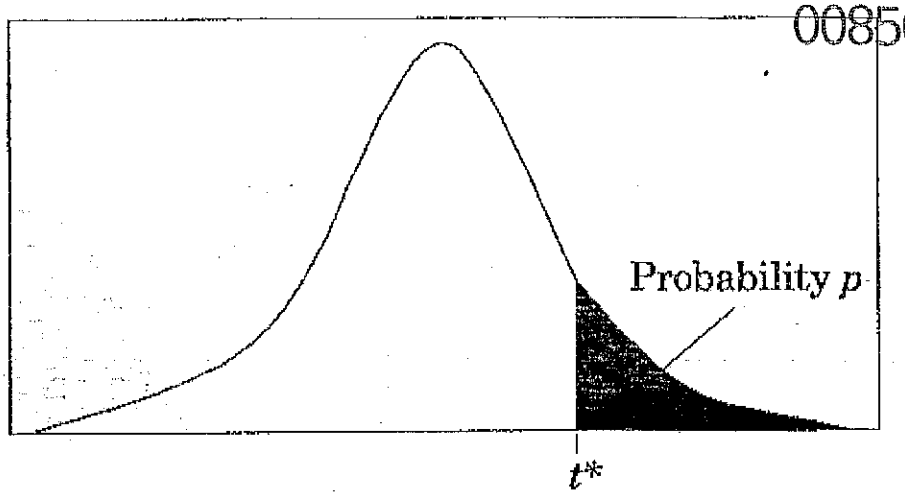
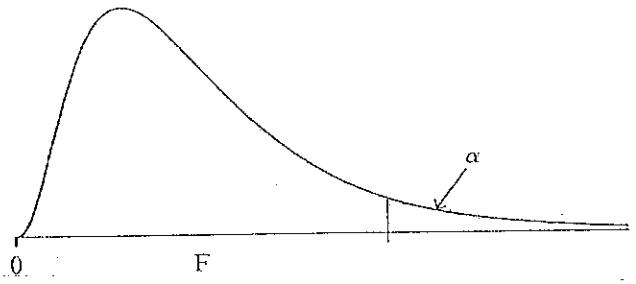


Table B

t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

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$\alpha = .05$

	df_1									
df_2	1	2	3	4	5	6	8	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	249.0	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.45	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.61	2.40
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50	2.30
13	4.67	3.80	3.41	3.18	3.02	2.92	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.38	2.20	2.00	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	1.98	1.73
25	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96	1.71
26	4.22	3.37	2.98	2.74	2.59	2.47	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.30	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.44	2.29	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.54	2.43	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.52	2.37	2.25	2.10	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.61	1.25
∞	3.84	2.99	2.60	2.37	2.21	2.09	1.94	1.75	1.52	1.00

Source: From Table V of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, published by Longman Group Ltd., London, 1974. (Previously published by Oliver & Boyd, Edinburgh.) Reprinted by permission of the authors and publishers.