THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY -LEVEL 05
FINAL EXAMINATION 2013/2014
MPJ 5134-The Nature of Mathematics I

Date: 11-8-2014 Time duration: 930hrs-1230hrs

Instructions:

- Answer only five(05) Questions
- No. of pages in the paper-03
- State clearly any assumptions you required
- All symbols are in standard notation unless they are defined

1. Let $a \in \mathbb{R}$. Prove the following:

   (a) $a = a + a.0$. [20%]

   (b) If $a \neq 0$ and $a.x = 1$ and $a.y = 1$, $x, y \in \mathbb{R}$ then $x = y$. [20%]

   (c) No rational number x in $\mathbb{Q}$ satisfies $x^2 = 2$. [20%]

   (d) If S is the set of all real numbers x such that $x^2 < 3$ then S is a non-empty bounded above set. [20%]

   (e) $\sqrt{(2)}$ is a real number. [20%]

2. (a) Let a and b be real numbers. Prove the following:

   i. $|a| \geq 0$ and $|a| = 0$ if and only if $a = 0$. [20%]

   ii. $|a+b| < |a| + |b|$. [30%]

   (b) At the XYZ software Company, the average starting salary for a new graphic designer is \$35,700, but the actual salary could differ from the average by as much \$3,520.

   i. Write an absolute value inequality to describe this situation. [20%]

   ii. Solve the inequality to find the range of the starting salaries. [30%]

3. (a) Let a,b,c be three integers. Prove the following:

   i. If $a|b$ and $b|c$ then $a|c$. [20%]

   ii. If $c|a$ and $c|b$, then $c|(xa + yb)$ for any integers x and y. [20%]

   iii. $a|1$ if and only if $a = \pm 1$. [20%]

   (b) Prove the following using Mathematical Induction.

   i. $2^{3n} - 1$ is a multiple of 7 for all n, where n is a natural number. [20%]

   ii. $P(n) : 10^n + 3.4^{n+2} + 5$ is divisible by 9. [20%]

4. (a) Let p be the statement "South West Monsoon is very good this year" and q be the statement "Rivers are rising", Give the verbal translation of statement (i) and verify the statement (ii).

   i. A. $p \vee \sim q$ and

   B. $\sim (\sim p \vee \sim q)$. [30%]

   ii. Statement $x > 1$ is false if and only if $x^2 > 1$ is false, when x is a real number. [20%]

   (b) With the help of truth tables, prove that

   i. $\sim (p \vee q) \equiv \sim p \wedge \sim q$. [25%]

   ii. $\sim (p \Rightarrow q) \equiv p \wedge \sim q \equiv \sim (q \vee \sim p)$. [25%]

5. Define what is meant by an argument $p_1, p_2, \ldots, p_n \vdash q$ is valid? [10%]

   (a) Test the validity of the following arguments:

      i. $p \Rightarrow q, (p \wedge q) \Rightarrow r \vdash q \Rightarrow r$. [25%]

      ii. if 7 is less than 4, then 7 is not a prime number. 7 is not less than 4. Therefore,7 is a prime number. [25%]

   (b) Let $p(x)$ be 'x is a mammal'. Let $q(x)$ be 'x is an animal'. Let $r(x)$ be 'x is warm blooded'.

      i. Translate into a formula: Every mammal is warm blooded. [20%]

      ii. Translate into English $(\exists x)(q(x) \wedge (\sim (p(x))))$. [20%]

6. (a) Let A, B, C be three sets. Prove or disprove the following using Venn diagram:

      i. $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$, [20%]

      ii. $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$. [20%]

   (b) Let A, B, C and D be sets. Prove the following results using either logic method or set properties:

      i. $C \subseteq A$ and $C \subseteq B \Leftrightarrow C \subseteq A \cap B$. [20%]

      ii. $(A \times B) \cup (C \times D) = \emptyset$ if and only if $A \cap C = \emptyset$ or $B \cap D = \emptyset$. [20%]

      iii. $(A \triangle B) \cap C = (A \cap C) \triangle (B \cap C)$. [20%]

7. Define the terms "equivalence Relation" and "equivalence class" on a set. [20%]

   (a) Let R be a binary relation defined as : $R = \{(a, b) \in \mathbb{R} : a - b \leq 3\}$. Determine whether R is reflexive, symmetric, anti-symmetric and transitive. [20%]

   (b) Let $A = \{1, 2, \ldots, 7\}$ and $R = \{(x, y) \in A : x - y$ is divisible by $3\}$. Show that R is an equivalence relation and hence find its equivalence classes. [20%]

   (c) Given integers a,b, with $b > 0$. Prove that there exists unique integers q and r satisfying $a = qb + r$, $0 \leq r \leq b$. [20%]

   (d) Let $a, b \neq 0$ be two integers. Prove that there exist integers x and y such that $\gcd(a, b) = ax + by$. [20%]