The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Diploma in Technology/Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX5233 Dynamics of Mechanical Systems**
Academic Year	:	2013/14
Date	:	11th August 2014
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

QUESTION 01

A machine mounted on the ground through an isolator consisting of a spring and a damper produces an excitation force. The transmissibility ratio (T) of the force transmitted to the ground at steady state is given, in usual notations, by

$$T = \left[\frac{1+4\beta^2 r^2}{\left(1-r^2\right)^2 + 4\beta^2 r^2}\right]^{1/2}$$

where, $r = \frac{\omega}{\omega_n}$, ω is the frequency of the excitation force, ω_n is the natural frequency of oscillation of the system and β is the damping ratio of the damper.

(a) Show that the transmissibility ratio is maximum when $r = \frac{1}{2\beta}\left[\sqrt{1+8\beta^2} - 1\right]^{1/2}$

(b) Determine, in terms of β, the transmissibility ratio at resonance.

(c) The following observations were made with respect to the performance of the above machine of mass 300 kg.

Transmissibility at resonance = $\sqrt{6}$

Transmissibility at an excitation frequency of 30 Hz = 0.5

Calculate the damping ratio of the damper and the stiffness of the spring.

QUESTION 02

Fig. Q2 shows a system of three rigid rods supported on springs, at mid points of rods. The stiffnesses of the springs are $2k$, k and k as indicated in the figure. One end of each rod is pivoted and the other end carries a mass m. Mass of the rods are negligible and the rods are horizontal in the equilibrium position. Assume that the motion of the system is confined to the plane of the paper.

(a) Determine the equations of motion for smaller angular oscillations.

(b) Show that the frequency equation can be expressed as,

$$(2\alpha - 1)(16\alpha^2 - 20\alpha + 1) = 0,$$

where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of angular oscillations of the system.

(c) If $m = 2$ kg and $k = 1000$ N/m, determine the natural frequencies.

(d) Determine the mode shape corresponding to the highest frequency.

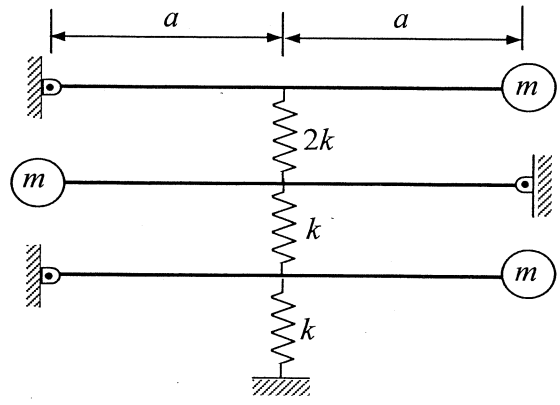


Fig. Q2

QUESTION 03

Fig. Q3 shows a rotor system consisting of three rotors of moments of inertia I, I and $2I$ and two shafts of torsional stiffnesses K and $2K$.

(a) Determine the equations of motion for small torsional oscillations.

(b) Show that the frequency equation can be expressed as,

$$\omega^2(\omega^2 - p)(\omega^2 - 4p) = 0,$$

where $p = \dfrac{K}{I}$ and ω is the natural frequency of torsional oscillations of the system.

(c) If I is equivalent to the moment of inertia of a rotor of mass 16 kg and diameter of 400 mm and K is equivalent to the torsional stiffness of a steel shaft of diameter 20 mm and length 500 mm, calculate the natural frequencies of torsional oscillations of the system.

Rigidity modulus of steel – 80 GN/m^2

(d) Explain what is meant by rigid body rotation.

(e) Sketch the shapes of possible modes of vibration of the system (No calculations are required).

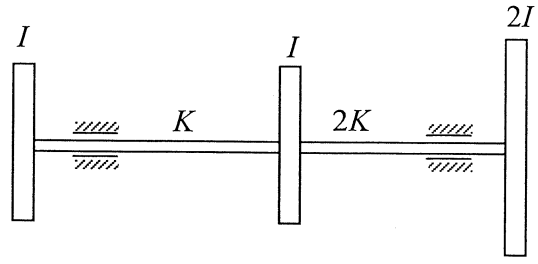


Fig. Q3

QUESTION 04

A uniform beam of mass m per unit length, flexural rigidity EI and length L is fixed at one end and simply supported at the other. Starting from the equation,

$$\frac{\partial^4 y}{\partial x^4} + \frac{m}{EI}\frac{\partial^2 y}{\partial t^2} = 0,$$

given in usual notations for the transverse vibration of a beam with constant cross section, show that the natural frequencies (ω) of transverse vibration of the beam may be obtained from the roots of the equation,

$$\tanh \lambda L - \tan \lambda L = 0, \text{ where } \lambda = \left(\frac{m\omega^2}{EI}\right)^{1/4}.$$

QUESTION 05

(a) Fig. Q5(a) shows a unity feed back control system.

(i) Determine the damped natural frequency and the settling time of this control system.

(ii) Determine the unit step response of this system. Assume all initial conditions to be zero.

(b) A controller is added to the above control system, as shown in Fig. Q5(b) in order to increase the damping ratio to 0.8.

(i) Determine the value of λ.

(ii) Determine the unit impulse response of the modified system. Assume all initial conditions to be zero.

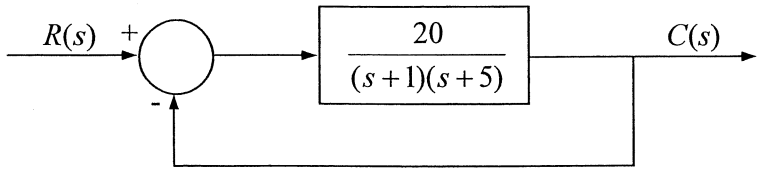


Fig. Q5(a)

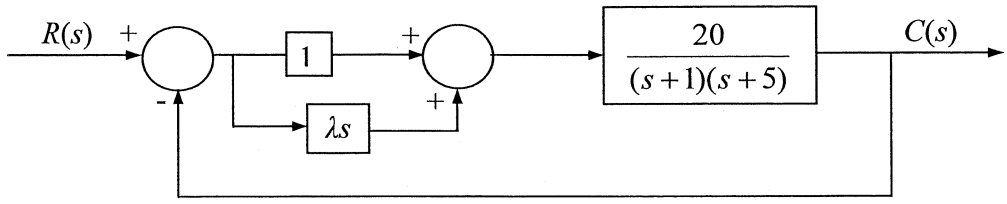


Fig. Q5(b)

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K}{(s+1)(s^2+6s+13)}$$

where K is the gain of the controller.

(a) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(b) Using the root locus you have drawn show that $s = -5$ is a closed loop pole of the system for $K = 32$. Determine the other closed loop poles.

(c) Determine the damping ratio of the closed loop system when $K = 32$.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s) = \frac{K}{s(s+3)(s+20)}.$$

(a) Draw either a Bode plot or a Nyquist plot for $K = 600$ and determine the gain margin and the phase margin of the closed loop control system.

(b) Determine the open loop gain for a,

(i) gain margin of 16 db

(ii) phase margin of 36^{o}

QUESTION 08

Fig. Q8 shows a hydraulic servomotor. A movement in the x direction of the valve causes to open passage 1 to the supply pressure, which in turn causes the big piston to move to the right. Because the sleeve is directly connected to this piston, the sleeve also moves to the right to close off flow from the valve. The flow into the big cylinder is proportional to the valve displacement with a proportionality constant k.

(a) Construct the block diagram for this system.

(b) Determine the transfer function relating the input position x to the output y. Identify the time constant.

(c) With the system initially at rest, x is given a step change of 10 mm. Obtain the variation of y with time and sketch a graph. Take $A = 1600$ mm^2 and $k = 3200$ mm^2.

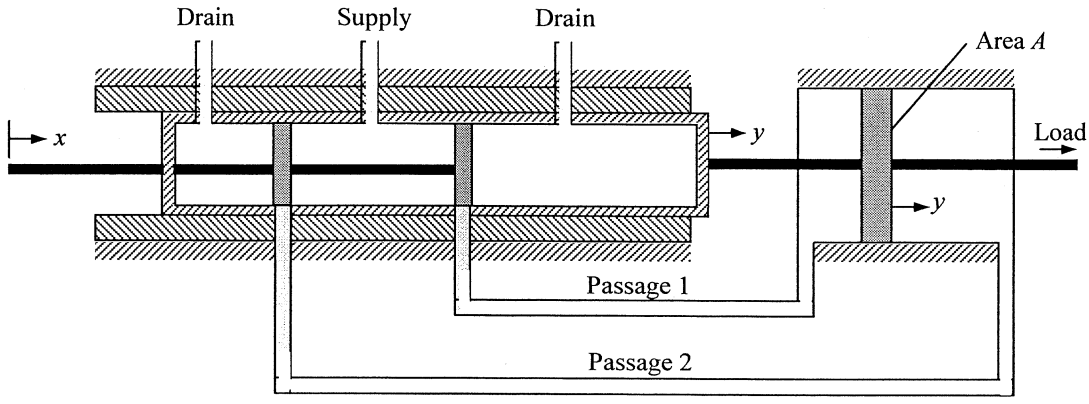


Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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