The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX5230 Fluid Mechanics**
Academic Year	:	2013/14
Date	:	September 03, 2014
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

Question 1

1.1 According to the internationally accepted conditions, standard atmosphere consists of a series of linearly decreasing, constant and increasing temperature zones.

At Troposphere, which is from sea level to an altitude (Z) of $11000m$, temperature (T) variation could be represented as, $\frac{dT}{dZ} = -m$, where m is the temperature lapse rate. At Stratosphere (altitude from $11000m$ to $20000m$) temperature remains constant.

Derive that, the pressure (P_1) at a point in the Troposphere, at which the temperature is T_1, can be expressed as, $$\frac{P_1}{P_0} = \left[\frac{T_1}{T_0}\right]^{\frac{g}{mR}},$$

and also show that, the pressure (P_2) at a point situated at an altitude of Z_2 in the Stratosphere can be derived as, $$\frac{P_2}{P_*} = e^{\left[-\frac{g}{RT_*}(Z_2 - Z_*)\right]}$$

where, P_0 and T_0 are pressure and temperature at sea level, P_* and T_* are pressure and temperature at an altitude Z_* (Z_*=$11000m$) and g is gravitational acceleration and R universal gas constant.

1.2 If the atmospheric sea level pressure and temperature are $101kPa$ and 20^oC respectively, and the lapse rate in the tropopause is $0.0075K/m$, calculate the temperature, pressure and density at $8{,}500m$ and at $12{,}500m$.

The tropopause is located at $11{,}000m$ and g= $9.81m/s^2$ and R=$287J/kgK$.

Question 2

2.1 Prove that for a body of fluid contained in a cylindrical vessel which rotates with uniform angular velocity of ω, the free surface of the fluid will be a parabolloid shape.

2.2 A vertical sluice gate, $2m$ wide and $1.2m$ height is shown in Figure.Q2. The gate is hinged at the bottom. There is a liquid, with a specific gravity of 1.45, on the upstream side of the gate which extends $1.5m$ above the top of the gate. On the downstream side there is water up to the top of the gate.

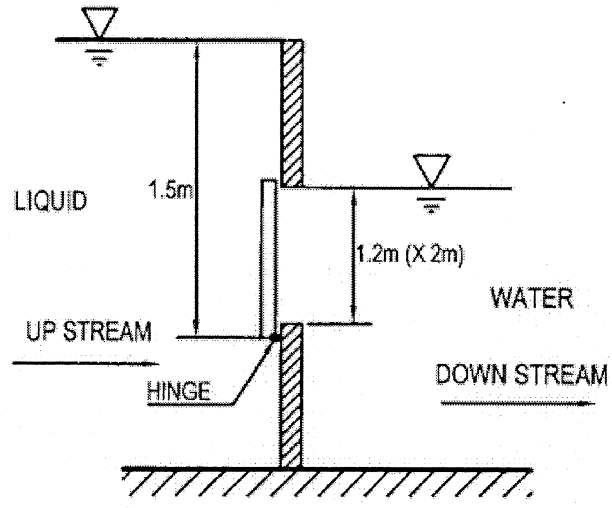


Figure.Q2

Find the,

i. resultant force acting on the gate.
ii. position of the centre of pressure.
iii. least force acting horizontally at the top of the gate which is capable of opening the gate.

Question 3

3.1 A cylinder is placed in a uniform flow, with a vortex of circulation Γ placed at the centre. If the complex potential of the flow is given by,

$\omega = Uz + U\frac{a^2}{z} + i\frac{\Gamma}{2\pi}lnz$ (in usual notations), calculate the lift acting on the cylinder.

3.2 To control the velocity of the wind in a large plot of plane land, it was decided to build a long vertical wind breaking wall at right angle to the direction of the wind. In order to reduce the velocity of the wind from $20m/s$ to $16m/s$ along a strip of land of $200m$ width directly in front of the wall, what would be the required height of the wall?

Question 4

Y-shaped part of a piping network is shown in Figure.Q4. Volumetric inflow at section 1-1 and outflow from section 3-3 are $12x10^{-3}m^3/s$ and $4x10^{-3}m^3/s$ respectively. Pressure gauge reading near section 1-1 is $25kPa$. The pipe diameters at sections 1-1, 2-2 and 3-3 are $100mm$, $80mm$ and $60mm$ respectively.

Determine the,

i. pressure at sections 2-2 and 3-3.
ii. force needed to hold the fitting in position.

Note: Assume one-dimensional flow and neglect elevation head and energy loss.

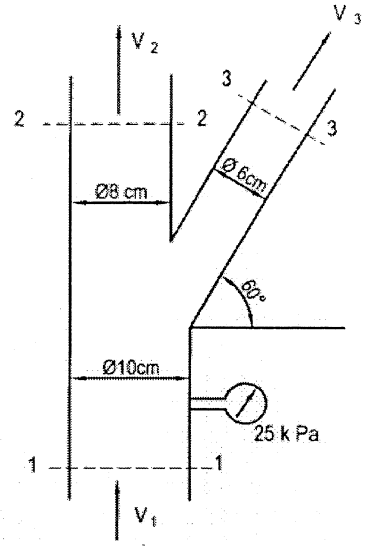


Figure.Q4

Question 5

5.1 Using Froude number as the criterion of dynamic similarity for a certain flow situation, find the scale factors for velocity, time, discharge, acceleration, force, work and power in terms of scale factor for length.

Note:

- *Froude number is,* $\frac{V}{\sqrt{Lg}}$, *where V is velocity, g is the gravitational acceleration and L is the length.*
- *Scale factor for length;* $L_r = \frac{L_m}{L_P}$ *, where* L_m *and* L_p *are the linear dimension of the model and the prototype respectively.*

5.2 A geometrically similar model of spillway is to be made to a scale of 1:50. Calculate the velocity ratio, discharge ratio and acceleration ratio.

5.3 If model prototype ratio is 1:75, show that the ratio of discharge per unit width of spillway can be given by $(1/75)^{3/2}$.

5.4 In the model test of a spillway, the discharge and the velocity of flow over the model were $2m^3/s$ and $1.5m/s$ respectively. Calculate the discharge and the velocity over the prototype which is 36 times the model size.

Question 6

6.1 Obtain a dimensionless variable out of density (ρ), coefficient of viscosity (μ), velocity (V) and diameter (D).

6.2 Show that the velocity (V) through an orifice is given by,

$$V = \sqrt{2gH}\, f\left[\frac{D}{H}, \frac{\mu}{\rho VH}, \frac{\sigma}{\rho V^2 H}\right]$$

where, H – head causing the flow, D – diameter of the orifice, μ – coefficient of viscosity, ρ – density, σ – surface tension and g - gravitational acceleration.

Question 7

7.1 Show that the loss of head (h_e) due to sudden expansion in a pipe line could be derived as a function of velocity head as,

$$h_e = \frac{[V_1 - V_2\ ]^2}{2g}$$

where, V_1 and V_2 are inlet and exit velocities respectively and g is gravitational accelerations.

7.2 A horizontal pipe section has a sudden contraction from 500*mm* to 250*mm* and following this contraction there is a sudden enlargement from 250*mm* to 500*mm*, as shown in Figure.Q7. Pressure variation at the contraction is from $103 \times 10^3 N/mm^2$ to $68 \times 10^3 N/mm^2$. Assuming the coefficient of contraction of fluid jet to be 0.65, Calculate,

i. rate of flow at the contraction.
ii. pressure at the end of expansion (section 3-3).

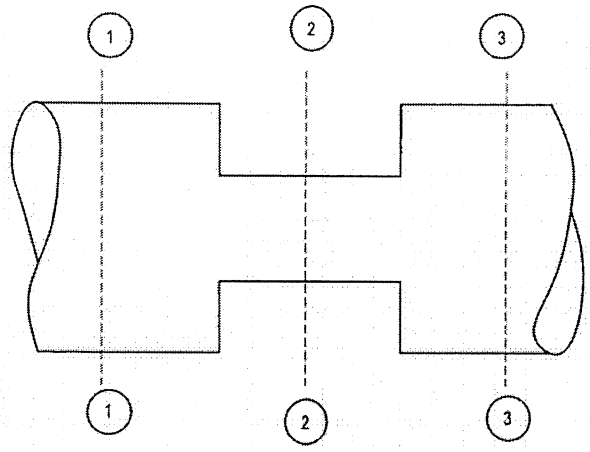


Figure.Q7

Question 8

Two concentric cylinders of radii R_1 and R_2, where ($R_2 > R_1$), are rotated at constant speeds ω_1 and ω_2 respectively. Starting from Navier–Stokes equations, show that for steady laminar flow between the two concentric cylinders, the tangential velocity V_θ at a radius r, where ($R_1 > r > R_2$), is given by,

$$V_\theta = \frac{1}{\left(R_2^2 - R_1^2\right)}\left[\left(R_2^2\omega_2 - R_1^2\omega_1\right)r + \frac{R_1^2 R_2^2\left(\omega_1 - \omega_2\right)}{r}\right]$$

Note:

- *Navier–Stokes equations in cylindrical co-ordinates for incompressible flow, in usual notation are given below.*

$$\rho\frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

$$\rho\frac{DVr}{Dt} = \frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 V_r}{\partial x^2} + \frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right)$$

$$\rho\frac{DV_\theta}{Dt} = \frac{1}{r}\frac{\partial p}{\partial_\theta} + \mu\left(\frac{\partial^2 V_\theta}{\partial x^2} + \frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right)$$

Where,

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V_r\frac{\partial u}{\partial r} + \frac{V_\theta}{r}\frac{\partial u}{\partial \theta}$$

$$\frac{DV_r}{Dt} = \frac{\partial V_r}{\partial t} + u\frac{\partial V_r}{\partial x} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial u}{\partial \theta} - \frac{V_\theta^2}{r}$$

$$\frac{DV_\theta}{Dt} = \frac{\partial V_\theta}{\partial t} + u\frac{\partial V_\theta}{\partial x} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} - \frac{V_r V_\theta}{r}$$

- *Continuing equation for incompressible fluid flow in terms of cylindrical co-ordinates is,*

$$\frac{\partial u}{\partial x} + \frac{\partial V_r}{\partial \gamma} + \frac{V_r}{\gamma} + \frac{1}{\gamma}\frac{\partial V_\theta}{\partial \theta} = 0$$

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