THE OPEN UNIVERSITY OF SRI LANKA

BACHLOR OF TECHNOLOGY – LEVEL 06

FINAL EXAMINATION - 2013/2014

MPJ6134 – THE NATURE OF MATHEMATICS II

Date: 13rd August 2014 **Time: 1330hrs –1630hrs**

Instructions:

- **Answer only five questions.**
- **Number of pages in the paper – 04.**
- **All symbols are in standard notation.**
- **If any results/ axioms/ theorems are required steps must be stated clearly.**

Questions

1. **(a)** A function $f: \mathbb{R} \longrightarrow (0,1)$ is defined by

$$f(x) = \frac{e^x}{1+e^x} \ , x \in R$$

A. Draw the graph of f. (2%)

B. Show that

i. f is one to one (4%)

ii. f is onto. (4%)

C. Find $f^{-1}(x)$. (4%)

(b) Functions g and h are defined by

$$g(x) = \begin{cases} x^2+1, & x \geq 0 \\ 2x+1, & x < 0 \end{cases} \text{ and } \ h(x) = 4x - 1 \ , x \in R$$

A. Draw the graph of h and g and hence determine the ranges of h and g. (2%)

B. Find $g \circ h(x)$ and $h \circ g(x)$. (4%)

P.T.O.

2. An axiomatic system for propositional calculus is denoted by L and it is defined as follows;

An axiom of L is any well formed formulas of one of the following the three a_1, a_2, a_3 forms.

For any well formed formulas $A, B,$ and C of L

a_1: $(A \Rightarrow (B \Rightarrow A))$

a_2: $\Big((A \Rightarrow (B \Rightarrow C)) \Rightarrow \big((A \Rightarrow B) \Rightarrow (A \Rightarrow C)\big)\Big)$

a_3: $\Big((\neg A \Rightarrow \neg B) \Rightarrow \big((\neg A \Rightarrow B) \Rightarrow A\big)\Big)$

(a) Show that a_1, a_2, a_3 are tautologies (3%)

(b) Using only the axioms, prove that

A. $\vdash A \Rightarrow A.$ (5%)

B. $\neg\neg A \vdash\Longrightarrow A$ deduce that $A \vdash\Longrightarrow \neg\neg A$ (12%)

3. (a) Let A and B be two propositions. Prove that

A. $\neg(A \Rightarrow B) \equiv A \wedge \neg B.$ (3%)

B. $\neg(A \wedge B) \equiv A \Rightarrow \neg B.$ (3%)

The statement Ψ is defined as follows (10%)

$$\Psi: \forall n\left(n \in \mathbb{N} \Rightarrow \exists m\left(m \in \mathbb{N} \wedge \forall x\left(0 < |x-a| < \frac{1}{m} \Rightarrow |f(x)-l| < \frac{1}{n}\right)\right)\right)$$

Using A and B find $\neg\Psi$ in terms of $\forall, \exists, \Rightarrow$ and $\wedge$.

(b). $B(x)$ means x is a boy. $G(x)$ means x is a girl. $L(x,y)$ means x loves y. $U(x)$ means x is unhappy.

A. Translate the following into everyday English; (2%)

$\exists x \exists y\big(G(x) \wedge B(y) \wedge L(x,y) \wedge \forall z(L(x,z) \Rightarrow z = y)\big).$

B. Translate the following sentences into well formed formulas (2%)

Every boy loves an exactly one girl.

4. Language of love is defined as follows $L = \{a, c, d, k, b(u), L(u,v), B(x),\ G(x)\}$ where $a =$ Amal, $c =$ Chamila, $d =$ Deshan, $k =$ Kumari, $b(u) =$ best friend of u, $L(u,v) =$ u loves v, $B(x) = x$ is a boy, $G(x) = x$ is a girl.

(a) Classify each of given in L (5%)

(b) Given that $\forall x\left(L(x,y) \Rightarrow \forall y\left(\exists u\left(L(z,b(u)) \wedge L(x,z)\right)\right)\right)$

A. Indicate the scope of each variable. (3%)

B. Indicate the free occurrence and bound occurrence of each variable. (6%)

C. State the nature of each variable. (3%)

(c) Write down three closed formulas (sentences) of L (3%)

5. Let L is the language having a binary relation symbol R and binary function symbol f.

(a) Which of the following are terms of L? Justify your answers.

A. $f(x, f(y,z))$ (2%)

B. $f((f(x,y),x)$ (2%)

(b) Which of the following are formulae of L? Justify your answers.

A. $\forall x \neg R(y,x)$ (2%)

B. $\forall x \neg R(z,y)$ (2%)

(c) Let M be the structure for L with $M = \mathbb{Z}^+ = \{1,\ 2, 3, \ldots..\}, f^M(n,m) = nm$, $R^M = \{\langle n,m \rangle \in M^2 : n < m\}$

A. Which of the following sentences of L are true in M?

i. $\forall x \forall y\left(R(x,y) \Rightarrow R(y,x)\right)$ (3%)

ii. $\exists x \forall y \neg\left(R(y, f(x,y))\right)$ (2%)

iii. $\forall x\left(R(x, f(x,x)) \Rightarrow \forall y R(y, f(x,y))\right)$ (3%)

B. Form the formulae $\varphi_1(x,y)$ and $\varphi_2(x,y)$ of L such that

i. $M \vDash \varphi_1(n,m) \Leftrightarrow n^2 < m$ (2%)

ii. $M \vDash \varphi_1(n,m) \Leftrightarrow n = m$ (2%)

6. State the langauge and axioms of Peano Arithmetic (4%)

(a) Using only axioms of Peano Arithmetic prove that

A. $\forall x(x = x)$ (5%)

B. $\forall x \forall y(x = y \Rightarrow y = x)$ (6%)

(b) Define number theoritic function. (2%)

Show that projection function is strongly representable in $\mathbb{N}$ (3%)

7. Explain the following topics.

(a) Godel's incompleteness theorems. (6%)

(b) Godel numbering. (9%)

Gven that Godel numbers of some primitive symbols as follows. Successor of x is denoted by $s(x)$.

0	s	=	$\neg$	$\vee$	$\forall$	(	)	+	$\times$	x
1	3	5	7	9	11	13	17	19	21	23

Find the Godel number of the following proof

$$S = \begin{Bmatrix} x = s(0) \\ \neg(s(0) = 0) \\ \neg(x = 0) \end{Bmatrix}$$ (5%)

End

Copyright Reserved