THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
POSTGRADUATE DIPLOMA IN TECHNOLOGY IN INDUSTRIAL ENGINEERING – LEVEL 7
FINAL EXAMINATION – 2011/2012
MEX 7211 – OPERATION RESEARCH
DATE : *22 March 2012*
TIME : *1400 hrs – 1700 hrs*
DURATION : *Three (03) hours*
ANSWER ANY FIVE QUESTIONS. ALL QUESTION CARRY EQUAL MARKS.
(Graph paper will be provided)

Q1. a. A coin is tossed twice. Since the probability of obtaining head is ½ in each toss, by the addition law, the probability of obtaining at least one head is (½ + ½) = 1. But this cannot be true, because we could get tail for both tosses. Explain this fallacy.

b. Dice is thrown twice and the sum of scores is calculated. Considering the sum of the scores to be a random variable, write down the probability distribution function for the sum of scores.

c. Due to turnover and absenteeism at an assembly plant, 20% of the items are assembled by inexperienced employees. Management has determined that customers return 12% of the items assembled by inexperienced employees, whereas only 5% of items assembled by experienced employees are returned.

(i) What is the probability that an item is returned?
(ii) Given that an item was returned, what is the probability that it was assembled by an inexperienced employee?

Q2. a. (i) Evaluate the expression ${}^{n}C_{r}p^{r}q^{(n-r)}$ where n=6, r=2 and p=0.7.

(ii) The probability that an industrial product will fail during the guarantee period is 0.15. In a batch of six of such industrial products, what is the probability that exactly two products will fail during the guarantee period?

b. (i) Evaluate the expression $e^{-a}\frac{a^{x}}{x!}$, where

a = 2, x =3 and e = 2.71.

(ii) A person on the average receives two nuisance calls per day. What is the probability that he will receive less than two nuisance calls on the next day?

c. The life time of a washing machine is normally distributed with a mean of 4 years. Only 15% of these machines last for at least five years.

(i) Calculate the standard deviation of life time of the washing machines.

d. A random sample of 36 tyres were selected and weighed. It was observed that the mean and standard deviation of the weights were 8 kg and 1.2 kg respectively. Develop a 95% confidence interval estimate for the mean weight.

Q3. The following marks have been obtained by eight students in the previous year.

Student Number	Assignment Test	Final Exam
	x	y
1	80	.75
2	40	46
3	50	52
4	60	58
5	64	62
6	52	48
7	56	55
8	82	80

(i) Calculate the correlation coefficient between assignment test marks and final exam marks.
(ii) Construct the line of regression $y = a + bx$
(iii) Estimate the final exam marks of a candidate who scored marks same as that at the assignment test.
(iv) Calculate the sum of squares error given by "SSE".
(v) Calculate the error of regression model given by S_{yx}.
(vi) Find the coefficient of determination "R^2"
(vii) Calculate the standard error of the "b" coefficient given by "S_b".

Q4. A businessman produce two types of bakery products type (A) and type (B). While all other resources are available in plenty, he has only 250 m of oven space and 600 kg flour for his daily production. One item of type (A) requires 1 m of oven space and 4 kg of flour. One item of type (B) requires 2 m of oven space and 3 kg flour. While the demand for type (A) is unlimited, the maximum daily demand for type (B) is 100. The businessman makes a profit of Rs. 30.00 on one item of type (A) and a profit of Rs. 100.00 on one item of type (B). He wishes to maximize profit of his business.

a. (i) Formulate the linear programming model.

(ii) Using graphical method find how many items of type (A) and type (B) he should produce to maximize profit.

b. It is now observed that by adding a certain ingredient to type (B) its quality is improved and the demand for type (B) would be unlimited. However the profit would reduce from Rs. 100.00 to Rs. 95.00. You need to evaluate this suggestion in the interest of the businessman. For this purpose.

(i) Formulate the new linear programming model.

(ii) Solve the problem graphically and advise the businessman.

Q5. Sri Lanka Metal Crafts Company makes brass trays and buckets. The number of trays (x_1) and buckets (x_2) that can be produced daily are subjected to the availability of brass and labour, which are the constraints. The company wishes to maximize their profit and the profit is denoted by "Z". The linear programming model is as follows.

Maximise $Z = 6x_1 + 10x_2$

$x_1 + 2x_2 \leq 90$ – Brass

$2x_1 + 2x_2 \leq 60$ - Labour

$x_1 \geq 0 \quad x_1 \geq 0$

The final table of the simplex solution which is incomplete is described below.

c_j / c_b	Basis	Solution	6	10		
			x_1	x_2	s_1	s_2
	x_2	20			1/3	-1/6
	x_1	10			-1/3	2/3
	z_j					
	$c_j - z_j$					

a. Copy this table and complete it.

b. Is the solution feasible? (give reasons)

c. Is the solution optimal? (give reasons)

d. Write down the optimal solution. (give reasons)

e. Are there multiple optimal solutions? (give reasons)

f. Find the range of the objective function coefficients (c_j values) for which the solution remains optimal.

g. Write down the dual of this problem.

h. By looking at the final simplex table of the primal, write down the optimal solution of the dual.

Q6. a. Briefly discuss the limitations of the assignment theory.

b. Four new machines M_1, M_2, M_3 and M_4 are to be installed in the machine shop of a newly built factory. Four locations L_1, L_2, L_3 and L_4 are reserved in the machine shop for the installation of these four the machines. The cost of installation of machines depends on the type of each machine and the location. The cost details of machine installation in different locations with respect to each machine are given below in the table.

Cost of installation (Rs. 000)

Machine	Location			
	L_1	L_2	L_3	L_4
M_1	9	11	18	12
M_2	12	9	14	10
M_3	7	11	14	11
M_4	12	8	12	7

Use assignment theory to find how the machines should be installed, so that the total cost of installation is a minimum.

Q7. a. Briefly explain the meaning of balanced transportation problem.

b. Sanjeewa company has three factories A, B and C that turns out gas cookers. The weekly capacities of the factories A, B and C are 5000, 6000 and 2500 respectively.

These cookers are transported to four distribution centers P, Q, R and S whose weekly demands are 6000, 4000, 2000 and 1500 respectively. The cost of transporting one unit from a given factory to a given distribution center is given in the table below.

Cost of Transport

Factory	Distribution center			
	P	Q	R	S
A	3	2	7	6
B	7	5	2	3
C	2	5	4	5

Sanjeewa company wishes to develop the transportation plan that would minimize total cost of transport.

(i) Find an initial feasible solution.
(ii) Solve the transportation problem.

Q8. An insurance company works 9 hours a day. They observe that insurance claims arrive in a poisson pattern at the rate of 4 per hour. These claims are evaluated by an officer who takes 10 minutes to evaluate one claim. This service time is observed to be negative exponential. The officer is helped by another person by way of searching for documents. Answer the following questions.

(i) What is the probability of having exactly five claims with the officer?
(ii) On the average how many claims are there with the insurance officer?
(iii) How long will a claim have to wait until it is attended to?
(iv) On the average how many claims are there waiting to be taken for evaluation?
(v) How long will a claim have to wait until it is taken for evaluation?
(vi) How many hours will the insurance officer idle per day?
(vii) Since the insurance officer has idle time, it is suggested that he could search for documents as well, and needs not have a helper. This arrangement without a helper would increase his service time to 18 minutes.
Do you agree with this suggestion? Briefly explain your answer with reasons.

Mathematical formulae

Queueing Theory

$P(n) = \theta^n (1-\theta)$

Server idle time $= H(1-\theta)$

$$L_s = \frac{\theta}{1-\theta}$$

$$L_Q = \frac{\theta^2}{1-\theta}$$

$L_s = \lambda W_s$

$L_Q = \lambda W_Q$

Correlation and regression

$$\frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]\left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}}$$

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \qquad a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$SSE = \sum y^2 - a\sum y - b\sum xy$$

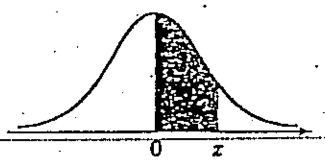
$$S_{yn} = \sqrt{\frac{SSE}{n-2}}$$

Coefficient of determination = r^2

$$S_b = \frac{S_{yx}}{SSx} \text{ where } SSx = \sum x^2 - \frac{(\sum x)^2}{n}$$

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t_:Îê19☻½u§$|˜â«µÏ♥.-4<☻àÞÎÎ!uô-þQi╚Ã☻§ß<Ê┌»U@BÒ0CW>‖j╣ΘÑ◄♥▬L1Ò(<77☻:Ð§EóÑ_o╣j

Normal Curve Areas



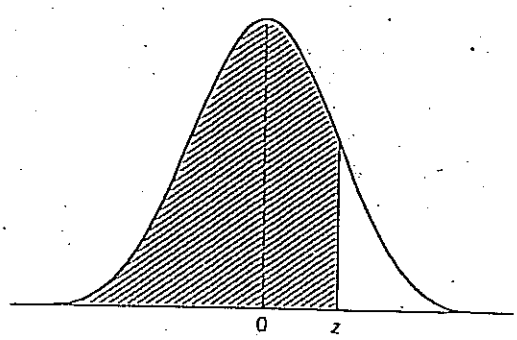
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

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Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $\phi(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.