The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	POSTGRADUATE DIPLOMA IN TECHNOLOGY IN INDUSTRIAL ENGINEERING
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX7211 OPERATIONS RESEARCH**
Academic Year	:	2012/13
Date	:	22nd August 2013
Time	:	0930 hours -1230 hours
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

Q1

(a) (i) A dice is thrown three times. What is the probability of getting 'six' in all three times?

(ii) How many times should a dice be thrown to be 95% sure of getting six at least once.

(b) A dice is thrown twice. E_1 is the event of getting "3" at the first throw. E_2 is the event where the sum of scores of the two throws is six, are the events E_1 and E_2 independent. (E_1 and E2 are independent if $P(E_1) = P(E_1/E_2)$)

(c) A market survey has revealed that 40% of the petrol in the market is contaminated and is of poor quality while remaining 60% is of good quality. It is estimated that the probability that a journey could be completed successfully when petrol is of good quality is 70% and when petrol is of poor quality is 20%

(i) What is the probability that a journey is completed successfully?

(ii) Given that the journey was completed successfully what is the probability that the petrol was of poor quality.

(iii) Given that the journey was not completed successfully what is the probability that the petrol was of good quality.

Q2

(a) (i) Evaluate the expression ${}^{n}C_{r}\, p^{r} q^{(n-r)}$ where n =8, r = 6 and P = 0.4.

(ii) Spark plugs are packed in boxes six in each box. Probability that a plug is defective is 0.2.

1) What is the probability that exactly two plugs are defective?

2) What is the probability that more than two plugs are defective?

(b) (i) Evaluate the expression $e^{-a}\frac{a^{x}}{x!}$ where a = 2, x = 3 and e = 2.71.

(ii) At a service station vehicles arrive on the average at the rate of two per hour.

1) What is the probability that three vehicles will arrive the next hour.

2) What is the probability that three vehicles will arrive within the next two hours.

(c) A motorist observes that the distance he could cover with one liter of petrol is normally distributed with mean 18 km and standard deviation 3 km.

(i) What is the probability that he could cover more than 21 km?

(ii) What is the probability that he covers more than 120 km with 5 liters of petrol?

(iii) How many liters of petrol should he pump to be 95% sure that he could cover a distance of 250 km.

NOTE If "x" is normally distributed with mean "m" and standard deviation "s" then "kx" is normally distributed with mean "km" and standard deviation "ks".

Q3.

A department store gives in service training to it's salesman followed by a test to consider whether they should terminate the services of any salesman who does not qualify the test. The following table gives the test scores and sales made by nine salesmen.

Test Score	14	19	24	21	28	22	15	20	19
Sales (Rs'000)	31	36	48	37	50	45	33	41	39

(i) Calculate the correlation coefficient between test scores and sales made

(ii) Develop the regression equation y = a + bx

(iii) If the firm wants a minimum sales volume of Rs. 30,000 what should the test score be to continue service without termination?

(iv) Calculate the sum of squares error given by SSE.

(v) Calculate – R^2 valve (coefficient of determination)

(vi) Calculate error of regression model given by S_{yx}.

(vii) Calculate the standard error of the "b" coefficient given by S_b.

Q4.

(a) Why do people resort to sampling instead of studying the full population?

(b) Briefly explain the terms "Sampling Error" and "Non Sampling Error".

(c) It is suggested that the life time of a car battery is normally distributed with mean 40 months and standard deviation 8 months.

(i) A study based on a sample is proposed to get a more accurate estimate of mean life time. What should the sample size be if the error should not exceed 2%.

(ii) A Sample of 100 batteries was examined and it was observed that the mean and standard deviation of life time are 50 months and 14 months respectively.

1) Develop a 95% confidence interval estimate of mean life time.

2) Develop a 80% confidence interval estimate of mean life time.

(iii) The manufacturer claims that the life time is 55 months. Carry out a statistical test to justify his claim. Use the hypothesis.
H_0 life time = 55 H_1 life time < 55 test at a level of significance of 5%.

Q5.

An oil company has two manufacturing units "A" and "B" which produce three different grades of oil namely super fine, medium and low grade. The company has to supply 12, 8 and 24 barrels of super fine, medium and low grade oils respectively per week. It costs the company Rs. 10,000 and Rs. 8,000 per day to run units "A" and "B" respectively. On each day unit "A" produces 6, 2 and 4 barrels and unit "B" produces 2, 2 and 12 barrels of super fine, medium and low grade oil respectively. The manager has to decide how many days per week should each unit be operated in order to minimize cost.

(i) Formulate the linear programming model.

(ii) Solve the problem using graphical method.

(iii) Suppose there is no demand for low grade oil and the requirement of 24 barrels is not there, would this change the optimal solution?

Q6.

The linear programming model of a maximization problem is as follows

$$\begin{aligned} \text{Maximize} \quad & z = 120x + 150y \\ & 3x + 3y \leq 180 \\ & 2x + 4y \leq 160 \\ & x > 0 \quad y > 0 \end{aligned}$$

The final simplex table of this problem is given below. However it is incomplete (it has blank cages)

c_b \ c_j	Basis	Solution	120	150		
			x	y	S_1	S_2
	x	40			$\frac{2}{3}$	$-\frac{1}{2}$
	y	20			$-\frac{1}{3}$	$\frac{1}{2}$
	z_j					
	$c_j - z_j$					

(i) Copy this table and complete it.
(ii) Is the solution feasible (Give reasons)
(iii) Is the solution optimal (Give reasons)
(iv) Are there multiple optimal solutions (Give Reasons)
(v) Write down the optimal solution.
(vi) Find the range of the objective function coefficient of "x" for which the solution remains optimal.
(vii) Write down the dual of this problem
(viii) By looking at the final simplex table of the primal, write down the optimal solution of the dual.

Q7

A businessman has four buses with different capacities and four route permits to run them. The daily profit earned would depend on the type of bus and on which route it runs on as explained in the table below. The businessman wishes to maximize profit.

(i) Use assignment theory to find how the buses should be assigned to routes so as to maximize profit.
(ii) Find how the buses should be assigned if it is given that bus B_1 can not be given route R_1.

Daily profit "Rs '000"

BUS	ROUTE			
	R_1	R_2	R_3	R_4
B_1	17	14	12	15
B_2	2	3	4	1
B_3	8	15	12	17
B_4	17	22	25	18

Q8

a) Prove that the condition for equilibrium in a single server queue is $\lambda < \mu$.
b) In a single server queue the average number of persons in the queuing system is 4. Find the probability that there are three units in the queuing system.

c) A health center has one operating theatre that works three shifts covering a period of 24 hours a day. The three shifts are served by three surgeons one surgeon for each shift. The mean time taken to perform one operation is 3 hours and has a negative exponential distribution. Patients arrive in a poison fashion at the rate of 7 per day. These patients are kept in a ward that is equipped with ten beds until they are called by the surgeon. The patients are put on a waiting list if ever the ward is full. The patients are served in the order they arrive.

(i) Evaluate the surgeon idle time per day.

(ii) On the average how many patients are there at the health center.

(iii) On the average how long must a patient wait until his treatment is complete at the health center?

(iv) What is the probability that the ward will be empty?

(v) What is the probability that a patient who just arrives will be put on the waiting list?

(vi) On the average how many patients will there be in the waiting list.

Mathematical formulae
Regression Analysis

$$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]\left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}}$$

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \quad a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$SSE = \sum y^2 - a\sum y - b\sum xy$$

$$SST = \sum y^2 - \frac{(\sum y)^2}{x}$$

$$R^2 = \left[1 - \frac{SSE}{SST}\right]$$

$$S_{yx} = \sqrt{\frac{SSE}{n-2}}$$

$$SSX = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$S_b = \frac{S_{yx}}{\sqrt{SSX}}$$

Queuing Theory

$P(n) = \theta^n(1-\theta)$

Server Idle time = H (1-θ)

$$L_s = \frac{\theta}{1-\theta}$$

$L_s = \lambda W_s$

$$1 + x + x^2 + \ldots\ldots + x^n + \text{--------} \infty = \frac{1}{1-x}$$

$$1 + 2x + 3x^2 + \ldots\ldots + nx^{n-1} + \text{--------} \infty = \frac{1}{(1-x)^2}$$

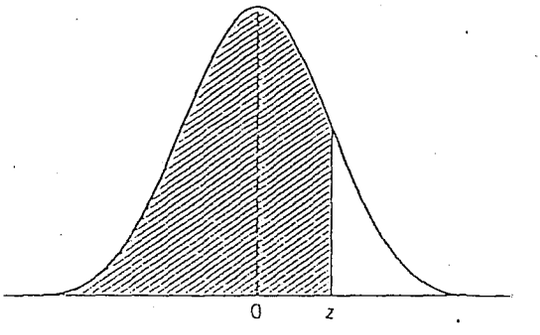
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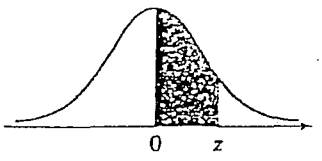
Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $\Phi(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.

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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

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